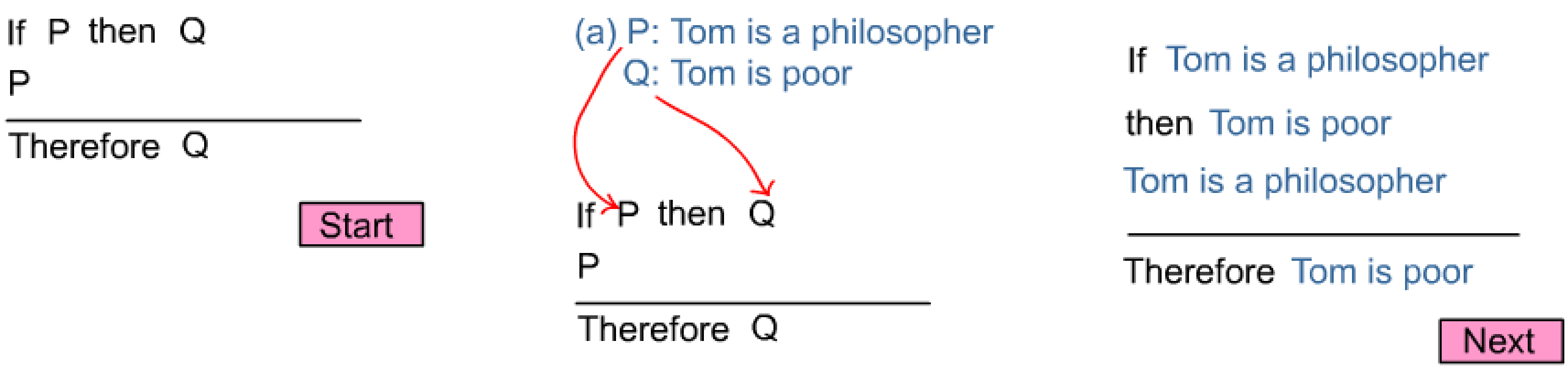
Symbolic Logic and Rules of Inference

http://philosophy.hku.hk/think/logic/whatislogic.php

If Tom is a philosopher, then Tom is poor.

Tom is a philosopher.

Therefore, Tom is poor.

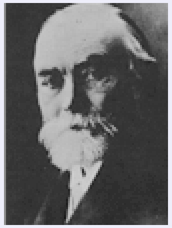


This is a valid form of argument, or Rule of Inference, called Modus Ponens.

Logic is topic neutral: rules can be used for multiple topics when the form is correct

§ L01.4 Topic neutrality

Modus ponens might be used to illustrate two features about the rules of reasoning in logic. The first feature is its topic-neutrality. As the four examples suggest, *modus ponens* can be used in reasoning about diverse topics. This is true of all the principles of reasoning in logic. The laws of biology might be true only of living creatures, and the laws of economics are only applicable to collections of agents that enagage in financial transactions. But the principles of logic are universal principles which are more general than biology and economics. This is in part what is implied in the following definitions of logic by two very famous logicians :



To discover truths is the task of all sciences; it falls to logic to discern the laws of truth. ... I assign to logic the task of discovering the laws of truth, not of assertion or thought." - Gottlob Frege (1848-1925)

From his 1956 paper "The Thought : A Logical Inquiry" in *Mind* Vol. 65.

"logic" ... [is] ... the name of a discipline which analyzes the meaning of the concepts common to all the sciences, and establishes the general laws governing the concepts. - Alfred Tarski (1901-1983)

From his *Introduction to logic and to the methodology of deductive sciences*, Dover, page xi.

Review of Well Formed Formulas: WFFs
http://philosophy.hku.hk/think/sl/wff.php

3. Are these expressions WFF?

- P Answer

  Yes. All sentence letters are WFFs.

- ~~~~~~~~~~(P&Q) Answer

  Yes.

- (P∨Q∨R) Answer

  No. Should be either (Pv(QvR)) or ((PvQ)vR)

- (~(P&S)) Answer

  No. An extra pair of brackets

- (~P) Answer

  No. An extra pair of brackets

- ((P↔Q)) Answer

  No. An extra pair of brackets

- ~(~G&~(~P&~Q)) Answer

  Yes.

Finding the main connective: http://philosophy.hku.hk/think/sl/wff.php

§ SL02.6 Main connective

The *main connective* in a WFF φ is the connective that has the widest scope. Here are some examples where the main connectives are highlighted in red:

1. **~**(P&Q)
2. **~**~~(P&Q)
3. **~**(~P&(P&Q))
4. (~(~P&Q)→P)
5. **~**(~(~P&Q)↔P)
6. ((~M&N)**&**R)
7. (~(~M&N)**&**R)

You will probably realize that we can use the main connective of a WFF to define whether it is a negation, a biconditional or a conditional, a disjunction or a conjunction.

Describing the parts of a well formed formula statement:

§ SL02.7 Exercises

Answer the following questions :

1. Which is the first conjunct of "((P&Q)&R)"? Answer

   (P&Q)

2. Which is the second disjunct of "((P∨Q)&R)"? Answer

   This is a trick question! The WFF is a conjunction so it does not have any disjuncts!

3. Which is the second disjunct of the first conjunct of "((P∨Q)&R)"? Answer

   Q

4. Suppose the antecedent of a conditional is a disjunction where both disjuncts are "P", and the consequent is the negation of the antecedent. What does this conditional look like? Answer

   ((PvP)->~(PvP))

5. Give an example of a conjunctive WFF where the first conjunct is the negation of the second conjunct. Answer

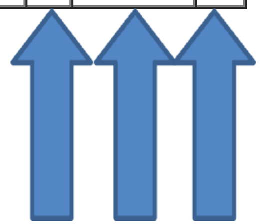
   (~P&P), or (~Q&Q), or (~(P&S)&(P&S)), etc.

Testing arguments for validity using truth tables:

- http://philosophy.hku.hk/think/sl/full.php

- P
- (P->Q)
- Therefore, Q

P	Q	P	(P→Q)	Q
T	T	T	T	**T**
T	F	T	F	**F**
F	T	F	T	**T**
F	F	F	T	**F**



Valid, no line has all premises true with a false conclusion.

Invalid argument

- Has at least one line where all premises are True and the conclusion is False:
- P->Q
- ~P
- Therefore ~Q

Remember that “(P→Q), ~P, therefore ~Q” is invalid. Verify for yourself that this is the truth-table for the sequent :

P	Q	(P→Q)	~P	~Q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

This is the invalid line, where the premises are both T and the conclusion is False.

http://www.butte.edu/~wmwu/iLogic/3.2/iLogic_3_2.html

- Truth Tables with automatic completion from Butte College in California

G ∨ (~*H* ⊃ *K*)

G ∨ (~*H* ⊃ *K*)
F ∨ (~T ⊃ F)
F ∨ (F ⊃ F)
F ∨ T
T

On this website, you can click each of the little buttons (they look like Play and Rewind on a remote control) to see how the truth table fills in!

That page also has special sections for single statements and pairs of statements

- **Single: Tautology, Contradiction, Contingent**
- **Pairs: Logically Equivalent, Logically Contradictory, Consistent and Inconsistent**

Self-contradiction

A statement is **self-contradictory** if it is logically false, that is, if it is logically impossible for the statement to be true. After completing the truth table of the conjunction *D* • ~*D*, we see that all the truth values in the main column under the dot are Fs. The truth table illustrates clearly that it is logically impossible for *D* • ~*D* to be true.

D • ~*D*

D	•	~	D
T			T
F			F

D	•	~	D
T		F	T
F		T	F

D	•	~	D
T	F	F	T
F	F	T	F

Review: Types of sentences part 1:

A is true.	A
A is false.	$\sim A$
A isn't so.	$\sim A$
Either *A* or *B* (or both).	$A \vee B$
A unless *B*.	$A \vee B$*
A or else *B*.	$A \vee B$
If *A*, then *B*.	$A \supset B$
A if *B*.	$B \supset A$

Review: Types of sentences part 2:

A only if *B*.	$A \supset B$
Only if *A*, *B*.	$B \supset A$
A if and only if *B*.	$A \equiv B$
A is a necessary condition for *B*.	$B \supset A$
A is a sufficient condition for *B*.	$A \supset B$
A necessary condition for A is *B*.	$A \supset B$
A sufficient condition of A is *B*.	$B \supset A$
A necessary and sufficient condition for *A* is *B*.	$A \equiv B$
A is a necessary and sufficient condition for *B*.	$A \equiv B$
Neither *A* nor B.	$\sim(A \vee B)$
Either not *A* or not *B*.	$\sim A \vee \sim B$
Neither not *A* nor not *B*.	$\sim(\sim A \vee \sim B)$
Both not *A* and not *B*.	$\sim A \cdot \sim B$
Not both *A* and *B*.	$\sim(A \cdot B)$

~ • ∨ ⊃ ≡

Truth Functions (how the operators are true or false)

p	$\sim p$
T	F
F	T

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Functions written out in English:

~ changes the truth value of whatever it has "scope" over from T to F and from F to T; note that some ~ are on letters and some are on brackets or parentheses.

• is only True if both sides are True, T•T.

V is only False if both sides are False, FvF. If there is even one T on one side, or T on both sides, the v is true.

⊃ is only false in a line where T⊃F, because that's like a cause happening without the effect. Any other combination of T⊃T, F⊃T, or F⊃F is T for the ⊃.

≡ is T as long as the sides match, T≡T or F≡F is True for the ≡.

Order of Operations ~ • v ⊃ ≡

- 1) Tilde ~ on single letters should be done right away, after the letters themselves are already done.
- 2) Connectors inside parentheses should be done next, after each side in the parentheses is done.
- 3) Tilde ~ outside of parentheses should be done next. Make sure you are changing the truth value of the connector column, not a side column.
- 4) Connectors that are in between two sets of parentheses should be done next.
- 5) Tilde ~ on big brackets, larger brackets [] and {}