5 | NEWTON'S LAWS OF MOTION

Figure 5.1 The Golden Gate Bridge, one of the greatest works of modern engineering, was the longest suspension bridge in the world in the year it opened, 1937. It is still among the 10 longest suspension bridges as of this writing. In designing and building a bridge, what physics must we consider? What forces act on the bridge? What forces keep the bridge from falling? How do the towers, cables, and ground interact to maintain stability?

Chapter Outline

Introduction

When you drive across a bridge, you expect it to remain stable. You also expect to speed up or slow your car in response to traffic changes. In both cases, you deal with forces. The forces on the bridge are in equilibrium, so it stays in place. In contrast, the force produced by your car engine causes a change in motion. Isaac Newton discovered the laws of motion that describe these situations.

Forces affect every moment of your life. Your body is held to Earth by force and held together by the forces of charged particles. When you open a door, walk down a street, lift your fork, or touch a baby’s face, you are applying forces. Zooming in deeper, your body’s atoms are held together by electrical forces, and the core of the atom, called the nucleus, is held together by the strongest force we know—strong nuclear force.

5.1 | Forces

Learning Objectives

By the end of the section, you will be able to:

- Distinguish between kinematics and dynamics
- Understand the definition of force
- Identify simple free-body diagrams
- Define the SI unit of force, the newton
- Describe force as a vector

The study of motion is called *kinematics*, but kinematics only describes the way objects move—their velocity and their acceleration. **Dynamics** is the study of how forces affect the motion of objects and systems. It considers the causes of motion of objects and systems of interest, where a system is anything being analyzed. The foundation of dynamics are the laws of motion stated by Isaac Newton (1642–1727). These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to situations on Earth and in space.

Newton's laws of motion were just one part of the monumental work that has made him legendary (**Figure 5.2**). The development of Newton's laws marks the transition from the Renaissance to the modern era. Not until the advent of modern physics was it discovered that Newton's laws produce a good description of motion only when the objects are moving at speeds much less than the speed of light and when those objects are larger than the size of most molecules (about 10^{-9} m in diameter). These constraints define the realm of Newtonian mechanics. At the beginning of the twentieth century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, quantum mechanics. Quantum mechanics does not have the constraints present in Newtonian physics. All of the situations we consider in this chapter, and all those preceding the introduction of relativity in **Relativity (http://cnx.org/content/m58555/latest/)** , are in the realm of Newtonian physics.

Figure 5.2 Isaac Newton (1642–1727) published his amazing work, *Philosophiae Naturalis Principia Mathematica*, in 1687. It proposed scientific laws that still apply today to describe the motion of objects (the laws of motion). Newton also discovered the law of gravity, invented calculus, and made great contributions to the theories of light and color.

Working Definition of Force

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. An intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or a pull has both magnitude and direction (therefore, it is a vector quantity), so we can define force as the push or pull on an object with a specific magnitude and direction. Force can be represented by vectors or expressed as a multiple of a standard force.

The push or pull on an object can vary considerably in either magnitude or direction. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea. Our everyday experiences also give us a good idea of how multiple forces add. If two people push in different directions on a third person, as illustrated in **Figure 5.3**, we might expect the total force to be in the direction shown. Since force is a vector, it adds just like other vectors. Forces, like other vectors, are represented by arrows and can be added using the familiar head-to-tail method or trigonometric methods. These ideas were developed in **Vectors**.

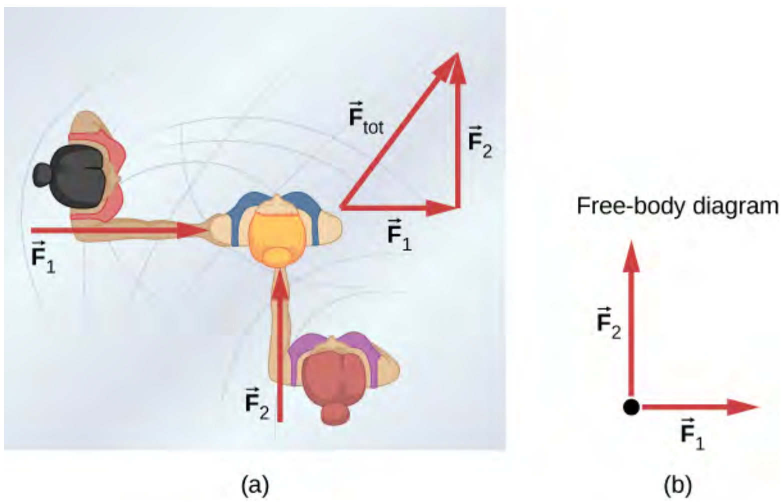


Figure 5.3 (a) An overhead view of two ice skaters pushing on a third skater. Forces are vectors and add like other vectors, so the total force on the third skater is in the direction shown. (b) A free-body diagram representing the forces acting on the third skater.

Figure 5.3(b) is our first example of a **free-body diagram**, which is a sketch showing all external forces acting on an object or system. The object or system is represented by a single isolated point (or free body), and only those forces acting *on* it that originate outside of the object or system—that is, **external forces**—are shown. (These forces are the only ones shown because only external forces acting on the free body affect its motion. We can ignore any internal forces within the body.) The forces are represented by vectors extending outward from the free body.

Free-body diagrams are useful in analyzing forces acting on an object or system, and are employed extensively in the study and application of Newton's laws of motion. You will see them throughout this text and in all your studies of physics. The following steps briefly explain how a free-body diagram is created; we examine this strategy in more detail in **Drawing Free-Body Diagrams**.

Problem-Solving Strategy: Drawing Free-Body Diagrams

1. Draw the object under consideration. If you are treating the object as a particle, represent the object as a point. Place this point at the origin of an *xy*-coordinate system.
2. Include all forces that act on the object, representing these forces as vectors. However, do not include the net force on the object or the forces that the object exerts on its environment.
3. Resolve all force vectors into *x*- and *y*-components.
4. Draw a separate free-body diagram for each object in the problem.

We illustrate this strategy with two examples of free-body diagrams (**Figure 5.4**). The terms used in this figure are explained in more detail later in the chapter.

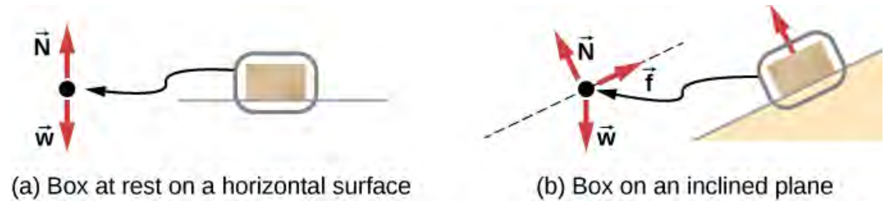


Figure 5.4 In these free-body diagrams, $\vec{\mathbf{N}}$ is the normal force, $\vec{\mathbf{w}}$ is the weight of the object, and $\vec{\mathbf{f}}$ is the friction.

The steps given here are sufficient to guide you in this important problem-solving strategy. The final section of this chapter explains in more detail how to draw free-body diagrams when working with the ideas presented in this chapter.

Development of the Force Concept

A quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard length. One possibility is to stretch a spring a certain fixed distance (**Figure 5.5**) and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be considered as multiples of this standard unit of force. Many other possibilities exist for standard forces. Some alternative definitions of force will be given later in this chapter.

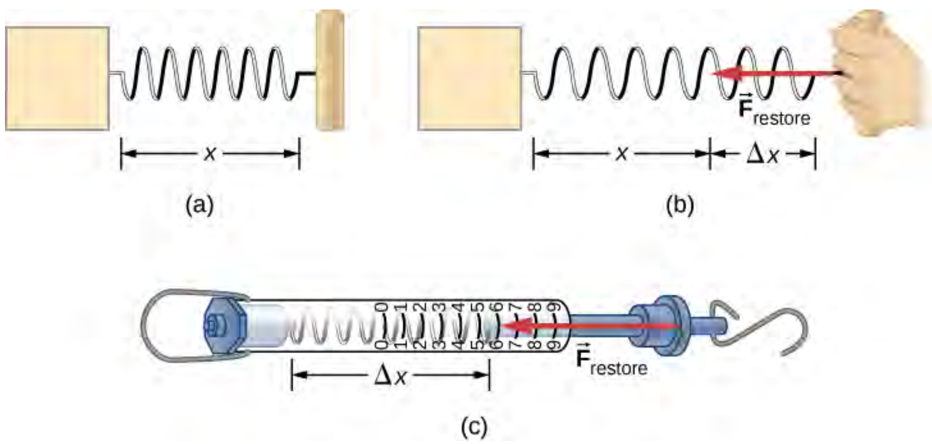


Figure 5.5 The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx, the spring exerts a restoring force $\vec{\mathbf{F}}_{\text{restore}}$, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\vec{\mathbf{F}}_{\text{restore}}$ is exerted on whatever is attached to the hook. Here, this force has a magnitude of six units of the force standard being employed.

Let's analyze force more deeply. Suppose a physics student sits at a table, working diligently on his homework (**Figure 5.6**). What external forces act on him? Can we determine the origin of these forces?

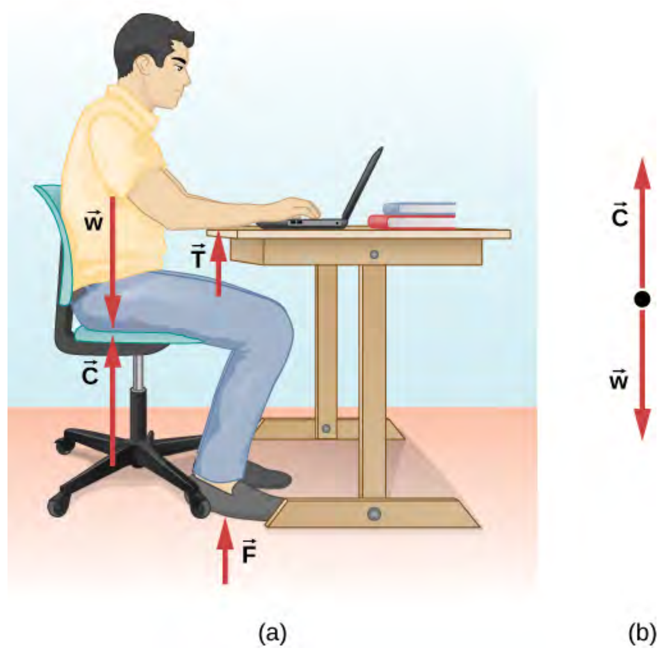


Figure 5.6 (a) The forces acting on the student are due to the chair, the table, the floor, and Earth's gravitational attraction. (b) In solving a problem involving the student, we may want to consider the forces acting along the line running through his torso. A free-body diagram for this situation is shown.

In most situations, forces are grouped into two categories: *contact forces* and *field forces*. As you might guess, contact forces are due to direct physical contact between objects. For example, the student in **Figure 5.6** experiences the contact forces $\vec{\mathbf{C}}$, $\vec{\mathbf{F}}$, and $\vec{\mathbf{T}}$, which are exerted by the chair on his posterior, the floor on his feet, and the table on his forearms, respectively. Field forces, however, act without the necessity of physical contact between objects. They depend on the presence of a "field" in the region of space surrounding the body under consideration. Since the student is in Earth's gravitational field, he feels a gravitational force $\vec{\mathbf{w}}$; in other words, he has weight.

You can think of a field as a property of space that is detectable by the forces it exerts. Scientists think there are only four fundamental force fields in nature. These are the gravitational, electromagnetic, strong nuclear, and weak fields (we consider these four forces in nature later in this text). As noted for $\vec{\mathbf{w}}$ in **Figure 5.6**, the gravitational field is responsible for the weight of a body. The forces of the electromagnetic field include those of static electricity and magnetism; they are also responsible for the attraction among atoms in bulk matter. Both the strong nuclear and the weak force fields are effective only over distances roughly equal to a length of scale no larger than an atomic nucleus (10^{-15} m). Their range is so small that neither field has influence in the macroscopic world of Newtonian mechanics.

Contact forces are fundamentally electromagnetic. While the elbow of the student in **Figure 5.6** is in contact with the tabletop, the atomic charges in his skin interact electromagnetically with the charges in the surface of the table. The net (total) result is the force $\vec{\mathbf{T}}$. Similarly, when adhesive tape sticks to a piece of paper, the atoms of the tape are intermingled with those of the paper to cause a net electromagnetic force between the two objects. However, in the context of Newtonian mechanics, the electromagnetic origin of contact forces is not an important concern.

Vector Notation for Force

As previously discussed, force is a vector; it has both magnitude and direction. The SI unit of force is called the **newton** (abbreviated N), and 1 N is the force needed to accelerate an object with a mass of 1 kg at a rate of $1\ \text{m/s}^2$: $1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$. An easy way to remember the size of a newton is to imagine holding a small apple; it has a weight of about 1 N.

We can thus describe a two-dimensional force in the form $\vec{\mathbf{F}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ (the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ indicate the direction of these forces along the *x*-axis and the *y*-axis, respectively) and a three-dimensional force in the form $\vec{\mathbf{F}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$. In **Figure 5.3**, let's suppose that ice skater 1, on the left side of the figure, pushes horizontally with a force of 30.0 N to the right; we represent this as $\vec{\mathbf{F}}_1 = 30.0\hat{\mathbf{i}}\text{ N}$. Similarly, if ice skater 2 pushes with a force of 40.0 N in the positive vertical direction shown, we would write $\vec{\mathbf{F}}_2 = 40.0\hat{\mathbf{j}}\text{ N}$. The resultant of the two forces causes a mass to accelerate—in this case, the third ice skater. This resultant is called the **net external force** $\vec{\mathbf{F}}_{\text{net}}$ and is found by taking the vector sum of all external forces acting on an object or system (thus, we can also represent net external force as $\sum \vec{\mathbf{F}}$):

$$\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots \tag{5.1}$$

This equation can be extended to any number of forces.

In this example, we have $\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 30.0\hat{\mathbf{i}} + 40.0\hat{\mathbf{j}}\text{ N}$. The hypotenuse of the triangle shown in **Figure 5.3** is the resultant force, or net force. It is a vector. To find its magnitude (the size of the vector, without regard to direction), we use the rule given in **Vectors**, taking the square root of the sum of the squares of the components:

$$F_{\text{net}} = \sqrt{(30.0\text{ N})^2 + (40.0\text{ N})^2} = 50.0\text{ N}.$$

The direction is given by

$$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right) = \tan^{-1}\left(\frac{40.0}{30.0}\right) = 53.1°,$$

measured from the positive *x*-axis, as shown in the free-body diagram in **Figure 5.3**(b).

Let's suppose the ice skaters now push the third ice skater with $\vec{\mathbf{F}}_1 = 3.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}\text{ N}$ and $\vec{\mathbf{F}}_2 = 5.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}\text{ N}$. What is the resultant of these two forces? We must recognize that force is a vector; therefore, we must add using the rules for vector addition:

$$\vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(3.0\hat{\mathbf{i}} + 8.0\hat{\mathbf{j}}\right) + \left(5.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}\right) = 8.0\hat{\mathbf{i}} + 12\hat{\mathbf{j}}\text{ N}$$

5.1 **Check Your Understanding** Find the magnitude and direction of the net force in the ice skater example just given.

View this **interactive simulation (https://openstaxcollege.org/l/21addvectors)** to learn how to add vectors. Drag vectors onto a graph, change their length and angle, and sum them together. The magnitude, angle, and components of each vector can be displayed in several formats.

5.2 | Newton's First Law

Learning Objectives

By the end of the section, you will be able to:

- Describe Newton's first law of motion
- Recognize friction as an external force
- Define inertia
- Identify inertial reference frames
- Calculate equilibrium for a system

Experience suggests that an object at rest remains at rest if left alone and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. However, **Newton's first law** gives a deeper explanation of this observation.

Newton's First Law of Motion

A body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force.

Note the repeated use of the verb "remains." We can think of this law as preserving the status quo of motion. Also note the expression "constant velocity;" this means that the object maintains a path along a straight line, since neither the magnitude nor the direction of the velocity vector changes. We can use Figure 5.7 to consider the two parts of Newton's first law.

Figure 5.7 (a) A hockey puck is shown at rest; it remains at rest until an outside force such as a hockey stick changes its state of rest; (b) a hockey puck is shown in motion; it continues in motion in a straight line until an outside force causes it to change its state of motion. Although it is slick, an ice surface provides some friction that slows the puck.

Rather than contradicting our experience, Newton's first law says that there must be a cause for any change in velocity (a change in either magnitude or direction) to occur. This cause is a net external force, which we defined earlier in the chapter. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappears, will the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface and ignoring air resistance, we can imagine the object sliding in a straight line indefinitely. Friction is thus the cause of slowing (consistent with Newton's first law). The object would not slow down if friction were eliminated.

Consider an air hockey table (**Figure 5.8**). When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object slows down.

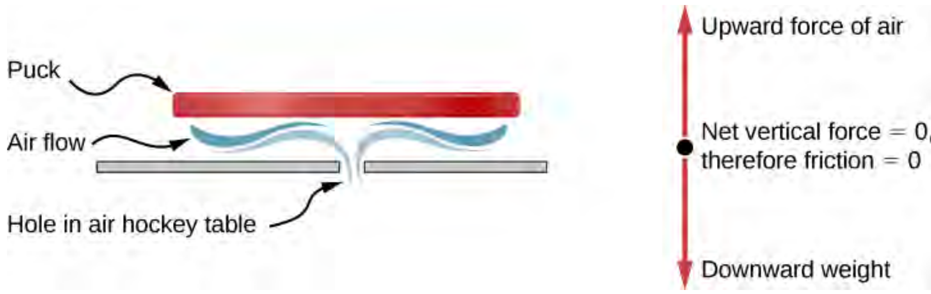


Figure 5.8 An air hockey table is useful in illustrating Newton's laws. When the air is off, friction quickly slows the puck; but when the air is on, it minimizes contact between the puck and the hockey table, and the puck glides far down the table.

Newton's first law is general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law of motion, and Newton, who clarified it, was to ask the fundamental question: "What is the cause?" Thinking in terms of cause and effect is fundamentally different from the typical ancient Greek approach, when questions such as "Why does a tiger have stripes?" would have been answered in Aristotelian fashion, such as "That is the nature of the beast." The ability to think in terms of cause and effect is the ability to make a connection between an observed behavior and the surrounding world.

Gravitation and Inertia

Regardless of the scale of an object, whether a molecule or a subatomic particle, two properties remain valid and thus of interest to physics: gravitation and inertia. Both are connected to mass. Roughly speaking, *mass* is a measure of the amount of matter in something. *Gravitation* is the attraction of one mass to another, such as the attraction between yourself and Earth that holds your feet to the floor. The magnitude of this attraction is your weight, and it is a force.

Mass is also related to **inertia**, the ability of an object to resist changes in its motion—in other words, to resist acceleration. Newton's first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is more difficult to change the motion of a large boulder than that of a basketball, for example, because the boulder has more mass than the basketball. In other words, the inertia of an object is measured by its mass. The relationship between mass and weight is explored later in this chapter.

Inertial Reference Frames

Earlier, we stated Newton's first law as "A body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force." It can also be stated as "Every body remains in its state of uniform motion in a straight line unless it is compelled to change that state by forces acting on it." To Newton, "uniform motion in a straight line" meant constant velocity, which includes the case of zero velocity, or rest. Therefore, the first law says that the velocity of an object remains constant if the net force on it is zero.

Newton's first law is usually considered to be a statement about reference frames. It provides a method for identifying a special type of reference frame: the **inertial reference frame**. In principle, we can make the net force on a body zero. If its velocity relative to a given frame is constant, then that frame is said to be inertial. So by definition, an inertial reference frame is a reference frame in which Newton's first law is valid. Newton's first law applies to objects with constant velocity. From this fact, we can infer the following statement.

Inertial Reference Frame

A reference frame moving at constant velocity relative to an inertial frame is also inertial. A reference frame accelerating relative to an inertial frame is not inertial.

Are inertial frames common in nature? It turns out that well within experimental error, a reference frame at rest relative to the most distant, or "fixed," stars is inertial. All frames moving uniformly with respect to this fixed-star frame are also inertial. For example, a nonrotating reference frame attached to the Sun is, for all practical purposes, inertial, because its velocity relative to the fixed stars does not vary by more than one part in 10^{10}. Earth accelerates relative to the fixed stars because it rotates on its axis and revolves around the Sun; hence, a reference frame attached to its surface is not inertial. For most problems, however, such a frame serves as a sufficiently accurate approximation to an inertial frame, because the acceleration of a point on Earth's surface relative to the fixed stars is rather small ($< 3.4 \times 10^{-2}$ m/s^2). Thus, unless indicated otherwise, we consider reference frames fixed on Earth to be inertial.

Finally, no particular inertial frame is more special than any other. As far as the laws of nature are concerned, all inertial frames are equivalent. In analyzing a problem, we choose one inertial frame over another simply on the basis of convenience.

Newton's First Law and Equilibrium

Newton's first law tells us about the equilibrium of a system, which is the state in which the forces on the system are balanced. Returning to **Forces** and the ice skaters in **Figure 5.3**, we know that the forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ combine to form a resultant force, or the net external force: $\vec{\mathbf{F}}_R = \vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$. To create equilibrium, we require a balancing force that will produce a net force of zero. This force must be equal in magnitude but opposite in direction to $\vec{\mathbf{F}}_R$, which means the vector must be $-\vec{\mathbf{F}}_R$. Referring to the ice skaters, for which we found $\vec{\mathbf{F}}_R$ to be $30.0\,\hat{\mathbf{i}} + 40.0\,\hat{\mathbf{j}}$ N, we can determine the balancing force by simply finding $-\vec{\mathbf{F}}_R = -30.0\,\hat{\mathbf{i}} - 40.0\,\hat{\mathbf{j}}$ N. See the free-body diagram in **Figure 5.3**(b).

We can give Newton's first law in vector form:

$$\vec{\mathbf{v}} = \text{constant when } \vec{\mathbf{F}}_{net} = \vec{\mathbf{0}} \text{ N.} \tag{5.2}$$

This equation says that a net force of zero implies that the velocity $\vec{\mathbf{v}}$ of the object is constant. (The word "constant" can indicate zero velocity.)

Newton's first law is deceptively simple. If a car is at rest, the only forces acting on the car are weight and the contact force of the pavement pushing up on the car (**Figure 5.9**). It is easy to understand that a nonzero net force is required to change the state of motion of the car. However, if the car is in motion with constant velocity, a common misconception is that the engine force propelling the car forward is larger in magnitude than the friction force that opposes forward motion. In fact, the two forces have identical magnitude.

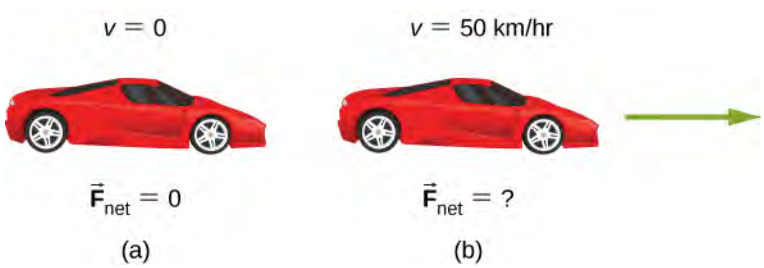


Figure 5.9 A car is shown (a) parked and (b) moving at constant velocity. How do Newton's laws apply to the parked car? What does the knowledge that the car is moving at constant velocity tell us about the net horizontal force on the car?

Example 5.1

When Does Newton's First Law Apply to Your Car?

Newton's laws can be applied to all physical processes involving force and motion, including something as mundane as driving a car.

(a) Your car is parked outside your house. Does Newton's first law apply in this situation? Why or why not?

(b) Your car moves at constant velocity down the street. Does Newton's first law apply in this situation? Why or why not?

Strategy

In (a), we are considering the first part of Newton's first law, dealing with a body at rest; in (b), we look at the second part of Newton's first law for a body in motion.

Solution

a. When your car is parked, all forces on the car must be balanced; the vector sum is 0 N. Thus, the net force is zero, and Newton's first law applies. The acceleration of the car is zero, and in this case, the velocity is also zero.

b. When your car is moving at constant velocity down the street, the net force must also be zero according to Newton's first law. The car's engine produces a forward force; friction, a force between the road and the tires of the car that opposes forward motion, has exactly the same magnitude as the engine force, producing the net force of zero. The body continues in its state of constant velocity until the net force becomes nonzero. Realize that *a net force of zero means that an object is either at rest or moving with constant velocity, that is, it is not accelerating.* What do you suppose happens when the car accelerates? We explore this idea in the next section.

Significance

As this example shows, there are two kinds of equilibrium. In (a), the car is at rest; we say it is in *static equilibrium*. In (b), the forces on the car are balanced, but the car is moving; we say that it is in *dynamic equilibrium*. (We examine this idea in more detail in **Static Equilibrium and Elasticity**.) Again, it is possible for two (or more) forces to act on an object yet for the object to move. In addition, a net force of zero cannot produce acceleration.

5.2 Check Your Understanding A skydiver opens his parachute, and shortly thereafter, he is moving at constant velocity. (a) What forces are acting on him? (b) Which force is bigger?

Engage this **simulation (https://openstaxcollege.org/l/21forcemotion)** to predict, qualitatively, how an external force will affect the speed and direction of an object's motion. Explain the effects with the help of a free-body diagram. Use free-body diagrams to draw position, velocity, acceleration, and force graphs, and vice versa. Explain how the graphs relate to one another. Given a scenario or a graph, sketch all four graphs.

5.3 | Newton's Second Law

Learning Objectives

By the end of the section, you will be able to:

- Distinguish between external and internal forces
- Describe Newton's second law of motion
- Explain the dependence of acceleration on net force and mass

Newton's second law is closely related to his first law. It mathematically gives the cause-and-effect relationship between force and changes in motion. Newton's second law is quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton’s second law as a simple equation that gives the exact relationship of force, mass, and acceleration, we need to sharpen some ideas we mentioned earlier.

Force and Acceleration

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is acceleration. Newton’s first law says that a net external force causes a change in motion; thus, we see that a *net external force causes nonzero acceleration*.

We defined external force in **Forces** as force acting on an object or system that originates outside of the object or system. Let’s consider this concept further. An intuitive notion of *external* is correct—it is outside the system of interest. For example, in **Figure 5.10**(a), the system of interest is the car plus the person within it. The two forces exerted by the two students are external forces. In contrast, an internal force acts between elements of the system. Thus, the force the person in the car exerts to hang on to the steering wheel is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton’s first law. (The internal forces cancel each other out, as explained in the next section.) Therefore, we must define the boundaries of the system before we can determine which forces are external. Sometimes, the system is obvious, whereas at other times, identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton’s laws. This concept is revisited many times in the study of physics.

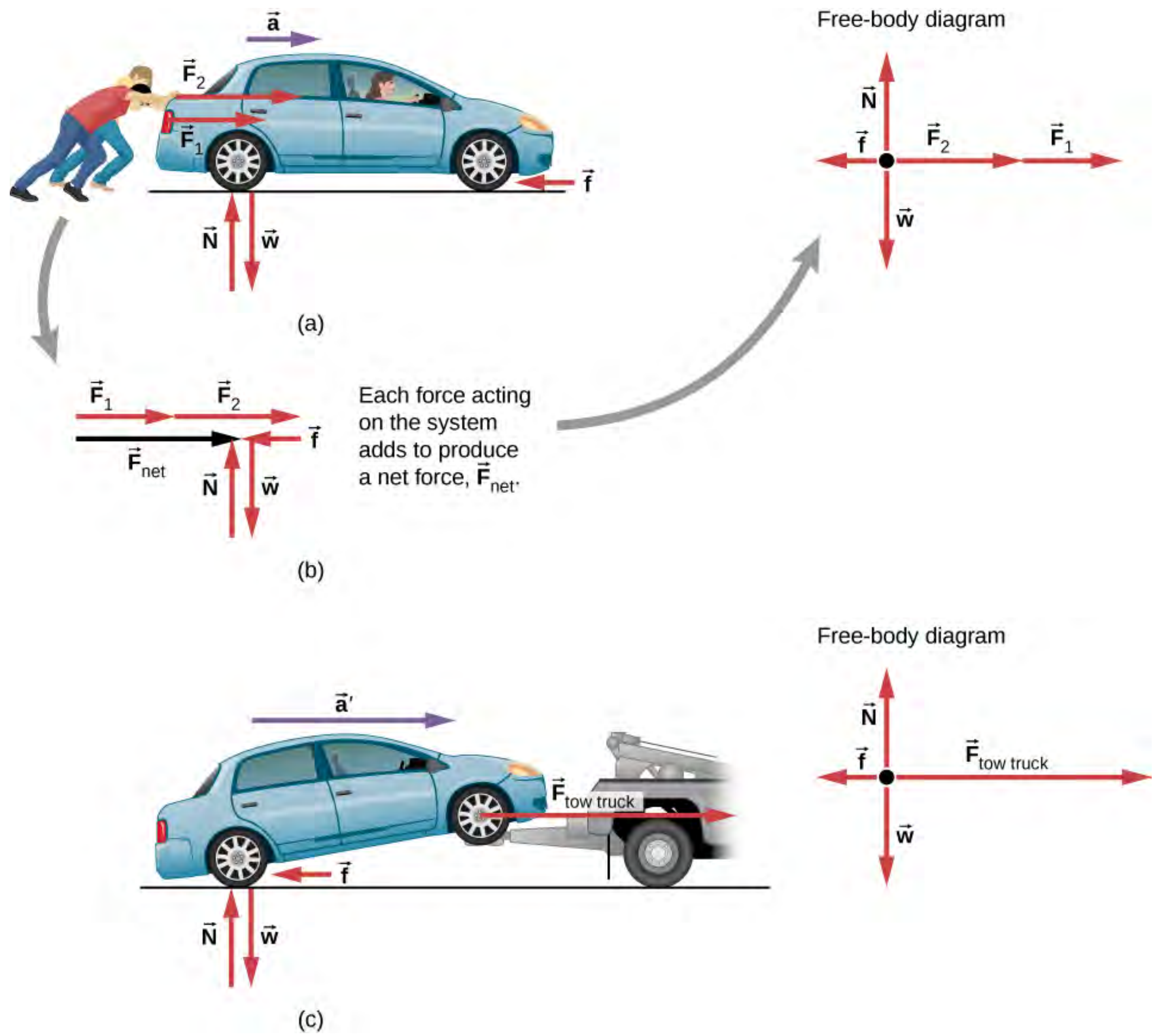


Figure 5.10 Different forces exerted on the same mass produce different accelerations. (a) Two students push a stalled car. All external forces acting on the car are shown. (b) The forces acting on the car are transferred to a coordinate plane (free-body diagram) for simpler analysis. (c) The tow truck can produce greater external force on the same mass, and thus greater acceleration.

From this example, you can see that different forces exerted on the same mass produce different accelerations. In **Figure 5.10**(a), the two students push a car with a driver in it. Arrows representing all external forces are shown. The system of interest is the car and its driver. The weight $\vec{\mathbf{w}}$ of the system and the support of the ground $\vec{\mathbf{N}}$ are also shown for completeness and are assumed to cancel (because there was no vertical motion and no imbalance of forces in the vertical direction to create a change in motion). The vector $\vec{\mathbf{f}}$ represents the friction acting on the car, and it acts to the left, opposing the motion of the car. (We discuss friction in more detail in the next chapter.) In **Figure 5.10**(b), all external forces acting on the system add together to produce the net force $\vec{\mathbf{F}}_{\text{net}}$. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, the vectors are shown collinearly. Finally, in **Figure 5.10**(c), a larger net external force produces a larger acceleration $(\vec{\mathbf{a}}' > \vec{\mathbf{a}})$ when the tow truck pulls the car.

It seems reasonable that acceleration would be directly proportional to and in the same direction as the net external force acting on a system. This assumption has been verified experimentally and is illustrated in **Figure 5.10**. To obtain an equation for Newton's second law, we first write the relationship of acceleration $\vec{\mathbf{a}}$ and net external force $\vec{\mathbf{F}}_{\text{net}}$ as the proportionality

$$\vec{\mathbf{a}} \propto \vec{\mathbf{F}}_{\text{net}}$$

where the symbol $\propto$ means "proportional to." (Recall from **Forces** that the net external force is the vector sum of all external forces and is sometimes indicated as $\sum \vec{\mathbf{F}}$.) This proportionality shows what we have said in words—acceleration is directly proportional to net external force. Once the system of interest is chosen, identify the external forces and ignore the internal ones. It is a tremendous simplification to disregard the numerous internal forces acting between objects within the system, such as muscular forces within the students' bodies, let alone the myriad forces between the atoms in the objects. Still, this simplification helps us solve some complex problems.

It also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. As illustrated in **Figure 5.11**, the same net external force applied to a basketball produces a much smaller acceleration when it is applied to an SUV. The proportionality is written as

$$a \propto \frac{1}{m},$$

where m is the mass of the system and a is the magnitude of the acceleration. Experiments have shown that acceleration is exactly inversely proportional to mass, just as it is directly proportional to net external force.

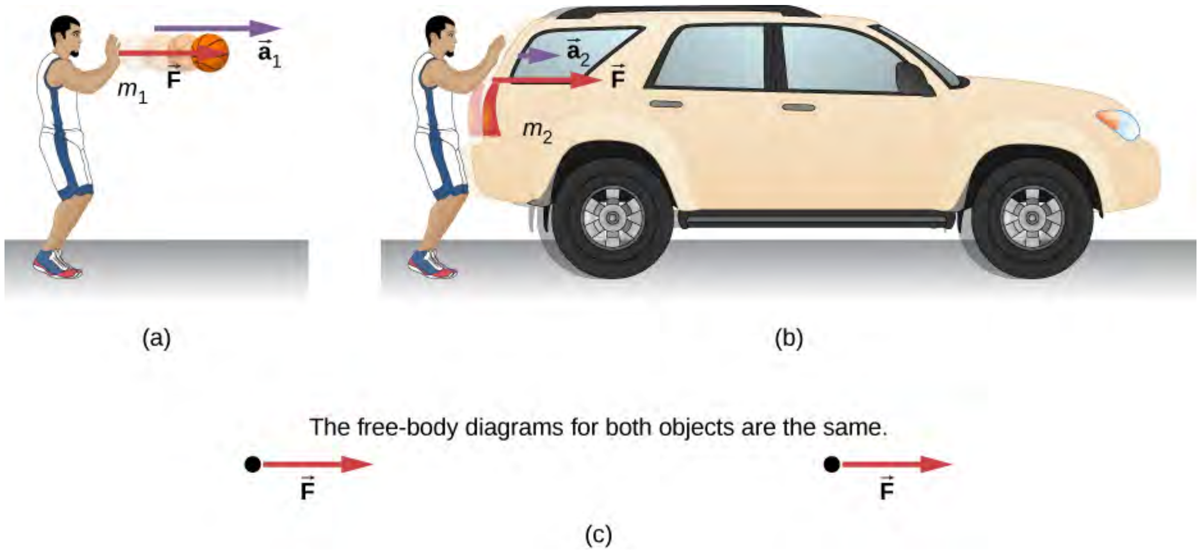


Figure 5.11 The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (Ignore the effect of gravity on the ball.) (b) The same player exerts an identical force on a stalled SUV and produces far less acceleration. (c) The free-body diagrams are identical, permitting direct comparison of the two situations. A series of patterns for free-body diagrams will emerge as you do more problems and learn how to draw them in **Drawing Free-Body Diagrams**.

It has been found that the acceleration of an object depends only on the net external force and the mass of the object. Combining the two proportionalities just given yields **Newton's second law**.

Newton's Second Law of Motion

The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportion to its mass. In equation form, Newton's second law is

$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m},$$

where $\vec{a}$ is the acceleration, $\vec{F}_{\text{net}}$ is the net force, and m is the mass. This is often written in the more familiar form

$$\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}, \tag{5.3}$$

but the first equation gives more insight into what Newton's second law means. When only the magnitude of force and acceleration are considered, this equation can be written in the simpler scalar form:

$$F_{\text{net}} = ma. \tag{5.4}$$

The law is a cause-and-effect relationship among three quantities that is not simply based on their definitions. The validity of the second law is based on experimental verification. The free-body diagram, which you will learn to draw in **Drawing Free-Body Diagrams**, is the basis for writing Newton's second law.

Example 5.2

What Acceleration Can a Person Produce When Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb.) parallel to the ground (**Figure 5.12**). The mass of the mower is 24 kg. What is its acceleration?

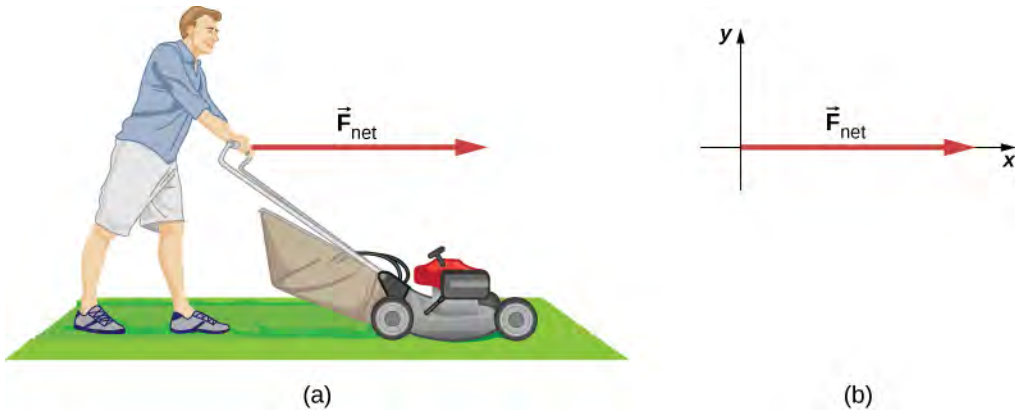


Figure 5.12 (a) The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right? (b) The free-body diagram for this problem is shown.

Strategy

This problem involves only motion in the horizontal direction; we are also given the net force, indicated by the single vector, but we can suppress the vector nature and concentrate on applying Newton's second law. Since F_{net} and *m* are given, the acceleration can be calculated directly from Newton's second law as $F_{net} = ma$.

Solution

The magnitude of the acceleration *a* is $a = F_{net}/m$. Entering known values gives

$$a = \frac{51\ \text{N}}{24\ \text{kg}}.$$

Substituting the unit of kilograms times meters per square second for newtons yields

$$a = \frac{51\ \text{kg} \cdot \text{m/s}^2}{24\ \text{kg}} = 2.1\ \text{m/s}^2.$$

Significance

The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. This is a result of the vector relationship expressed in Newton's second law, that is, the vector representing net force is the scalar multiple of the acceleration vector. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion (since we know the mower moved forward), and the vertical forces must cancel because no acceleration occurs in the vertical direction (the mower is moving only horizontally). The acceleration found is small enough to be reasonable for a person pushing a mower. Such an effort would not last too long, because the person's top speed would soon be reached.

5.3 Check Your Understanding At the time of its launch, the HMS *Titanic* was the most massive mobile object ever built, with a mass of 6.0×10^7 kg . If a force of 6 MN $(6 \times 10^6\ \text{N})$ was applied to the ship, what acceleration would it experience?

In the preceding example, we dealt with net force only for simplicity. However, several forces act on the lawn mower. The weight $\vec{\mathbf{w}}$ (discussed in detail in **Mass and Weight**) pulls down on the mower, toward the center of Earth; this produces a contact force on the ground. The ground must exert an upward force on the lawn mower, known as the normal force $\vec{\mathbf{N}}$, which we define in **Common Forces**. These forces are balanced and therefore do not produce vertical acceleration. In the next example, we show both of these forces. As you continue to solve problems using Newton's second law, be sure to show multiple forces.

Example 5.3

Which Force Is Bigger?

(a) The car shown in Figure 5.13 is moving at a constant speed. Which force is bigger, $\vec{\mathbf{F}}_{\text{engine}}$ or $\vec{\mathbf{F}}_{\text{friction}}$? Explain.

(b) The same car is now accelerating to the right. Which force is bigger, $\vec{\mathbf{F}}_{\text{engine}}$ or $\vec{\mathbf{F}}_{\text{friction}}$? Explain.

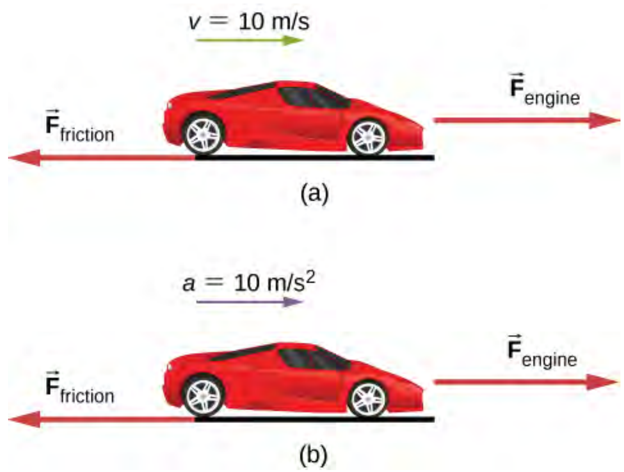


Figure 5.13 A car is shown (a) moving at constant speed and (b) accelerating. How do the forces acting on the car compare in each case? (a) What does the knowledge that the car is moving at constant velocity tell us about the net horizontal force on the car compared to the friction force? (b) What does the knowledge that the car is accelerating tell us about the horizontal force on the car compared to the friction force?

Strategy

We must consider Newton's first and second laws to analyze the situation. We need to decide which law applies; this, in turn, will tell us about the relationship between the forces.

Solution

a. The forces are equal. According to Newton's first law, if the net force is zero, the velocity is constant.

b. In this case, $\vec{\mathbf{F}}_{\text{engine}}$ must be larger than $\vec{\mathbf{F}}_{\text{friction}}$. According to Newton's second law, a net force is required to cause acceleration.

Significance

These questions may seem trivial, but they are commonly answered incorrectly. For a car or any other object to move, it must be accelerated from rest to the desired speed; this requires that the engine force be greater than the friction force. Once the car is moving at constant velocity, the net force must be zero; otherwise, the car will accelerate (gain speed). To solve problems involving Newton's laws, we must understand whether to apply Newton's first law (where $\sum \vec{\mathbf{F}} = \vec{\mathbf{0}}$) or Newton's second law (where $\sum \vec{\mathbf{F}}$ is not zero). This will be apparent as you see more examples and attempt to solve problems on your own.

Example 5.4

What Rocket Thrust Accelerates This Sled?

Before manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets.

Calculate the magnitude of force exerted by each rocket, called its thrust *T*, for the four-rocket propulsion system shown in **Figure 5.14**. The sled's initial acceleration is 49 m/s^2, the mass of the system is 2100 kg, and the force of friction opposing the motion is 650 N.

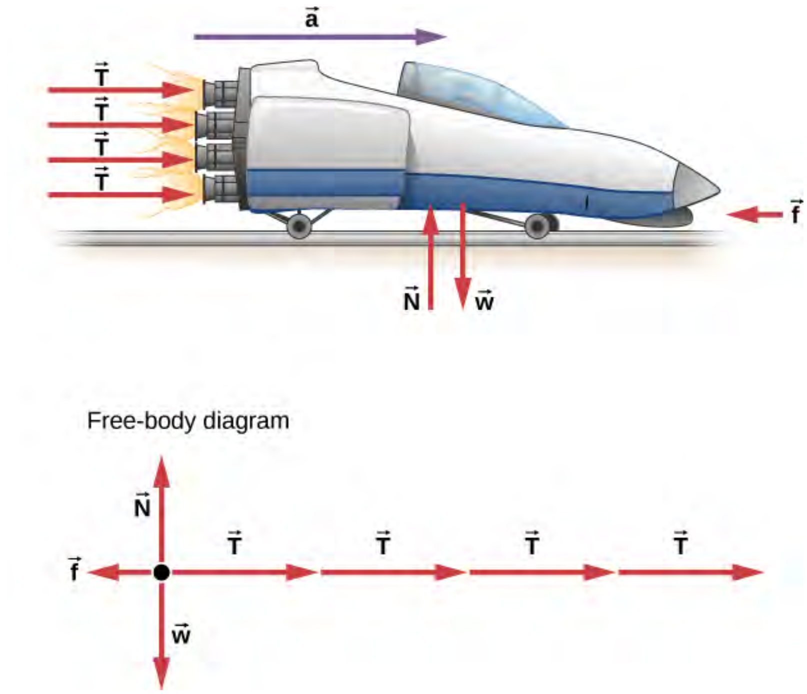


Figure 5.14 A sled experiences a rocket thrust that accelerates it to the right. Each rocket creates an identical thrust *T*. The system here is the sled, its rockets, and its rider, so none of the forces between these objects are considered. The arrow representing friction ($\vec{\mathbf{f}}$) is drawn larger than scale.

Strategy

Although forces are acting both vertically and horizontally, we assume the vertical forces cancel because there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in **Figure 5.14**.

Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. We have defined the direction of the force and acceleration as acting "to the right," so we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

$$F_{\text{net}} = ma$$

where F_{net} is the net force along the horizontal direction. We can see from the figure that the engine thrusts add, whereas friction opposes the thrust. In equation form, the net external force is

$$F_{\text{net}} = 4T - f.$$

Substituting this into Newton's second law gives us

$$F_{\text{net}} = ma = 4T - f.$$

Using a little algebra, we solve for the total thrust $4T$:

$$4T = ma + f.$$

Substituting known values yields

$$4T = ma + f = (2100\,\text{kg})\left(49\,\text{m/s}^2\right) + 650\,\text{N}.$$

Therefore, the total thrust is

$$4T = 1.0 \times 10^5\ \text{N},$$

and the individual thrusts are

$$T = \frac{1.0 \times 10^5\ \text{N}}{4} = 2.5 \times 10^4\ \text{N}.$$

Significance

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance, and the setup was designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 g's. (Recall that g, acceleration due to gravity, is $9.80\,\text{m/s}^2$. When we say that acceleration is 45 g's, it is $45 \times 9.8\,\text{m/s}^2$, which is approximately $440\,\text{m/s}^2$.) Although living subjects are not used anymore, land speeds of 10,000 km/h have been obtained with a rocket sled.

In this example, as in the preceding one, the system of interest is obvious. We see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.

Newton's second law is more than a definition; it is a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature.

5.4 Check Your Understanding A 550-kg sports car collides with a 2200-kg truck, and during the collision, the net force on each vehicle is the force exerted by the other. If the magnitude of the truck's acceleration is $10\,\text{m/s}^2$, what is the magnitude of the sports car's acceleration?

Component Form of Newton's Second Law

We have developed Newton's second law and presented it as a vector equation in **Equation 5.3**. This vector equation can be written as three component equations:

$$\sum \vec{\mathbf{F}}_x = m\vec{\mathbf{a}}_x,\ \sum \vec{\mathbf{F}}_y = m\vec{\mathbf{a}}_y,\ \text{and} \sum \vec{\mathbf{F}}_z = m\vec{\mathbf{a}}_z. \tag{5.5}$$

The second law is a description of how a body responds mechanically to its environment. The influence of the environment is the net force $\vec{\mathbf{F}}_{\text{net}}$, the body's response is the acceleration $\vec{\mathbf{a}}$, and the strength of the response is inversely proportional to the mass m. The larger the mass of an object, the smaller its response (its acceleration) to the influence of the environment (a given net force). Therefore, a body's mass is a measure of its inertia, as we explained in **Newton's First Law**.

Example 5.5

Force on a Soccer Ball

A 0.400-kg soccer ball is kicked across the field by a player; it undergoes acceleration given by $\vec{\mathbf{a}} = 3.00\,\hat{\mathbf{i}} + 7.00\,\hat{\mathbf{j}}\ \text{m/s}^2$. Find (a) the resultant force acting on the ball and (b) the magnitude and direction of the resultant force.

Strategy

The vectors in $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ format, which indicate force direction along the *x*-axis and the *y*-axis, respectively, are involved, so we apply Newton's second law in vector form.

Solution

a. We apply Newton's second law:

$$\vec{\mathbf{F}}_{\text{net}} = m\,\vec{\mathbf{a}} = (0.400\ \text{kg})\left(3.00\,\hat{\mathbf{i}} + 7.00\,\hat{\mathbf{j}}\ \text{m/s}^2\right) = 1.20\,\hat{\mathbf{i}} + 2.80\,\hat{\mathbf{j}}\ \text{N}.$$

b. Magnitude and direction are found using the components of $\vec{\mathbf{F}}_{\text{net}}$:

$$F_{\text{net}} = \sqrt{(1.20\ \text{N})^2 + (2.80\ \text{N})^2} = 3.05\ \text{N and } \theta = \tan^{-1}\left(\frac{2.80}{1.20}\right) = 66.8°.$$

Significance

We must remember that Newton's second law is a vector equation. In (a), we are multiplying a vector by a scalar to determine the net force in vector form. While the vector form gives a compact representation of the force vector, it does not tell us how "big" it is, or where it goes, in intuitive terms. In (b), we are determining the actual size (magnitude) of this force and the direction in which it travels.

Example 5.6

Mass of a Car

Find the mass of a car if a net force of $-600.0\,\hat{\mathbf{j}}\ \text{N}$ produces an acceleration of $-0.2\,\hat{\mathbf{j}}\ \text{m/s}^2$.

Strategy

Vector division is not defined, so $m = \vec{\mathbf{F}}_{\text{net}}/\vec{\mathbf{a}}$ cannot be performed. However, mass *m* is a scalar, so we can use the scalar form of Newton's second law, $m = F_{\text{net}}/a$.

Solution

We use $m = F_{\text{net}}/a$ and substitute the magnitudes of the two vectors: $F_{\text{net}} = 600.0\ \text{N}$ and $a = 0.2\ \text{m/s}^2$. Therefore,

$$m = \frac{F_{\text{net}}}{a} = \frac{600.0\ \text{N}}{0.2\ \text{m/s}^2} = 3000\ \text{kg}.$$

Significance

Force and acceleration were given in the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ format, but the answer, mass *m*, is a scalar and thus is not given in $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ form.

Example 5.7

Several Forces on a Particle

A particle of mass $m = 4.0\,\text{kg}$ is acted upon by four forces of magnitudes. $F_1 = 10.0\,\text{N},\ F_2 = 40.0\,\text{N},\ F_3 = 5.0\,\text{N},$ and $F_4 = 2.0\,\text{N}$, with the directions as shown in the free-body diagram in **Figure 5.15**. What is the acceleration of the particle?

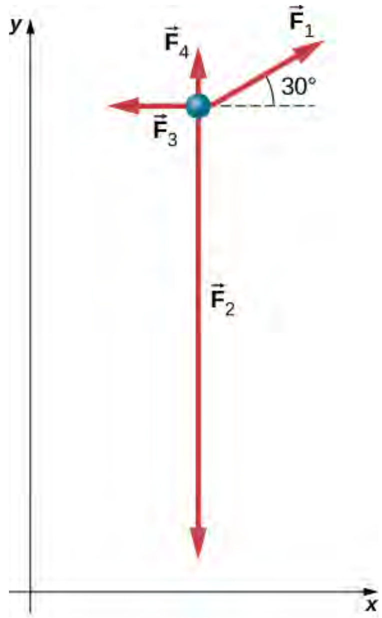


Figure 5.15 Four forces in the *xy*-plane are applied to a 4.0-kg particle.

Strategy

Because this is a two-dimensional problem, we must use a free-body diagram. First, $\vec{\mathbf{F}}_1$ must be resolved into *x*- and *y*-components. We can then apply the second law in each direction.

Solution

We draw a free-body diagram as shown in **Figure 5.15**. Now we apply Newton's second law. We consider all vectors resolved into *x*- and *y*-components:

$$\begin{aligned} &\sum F_x = ma_x \\ &F_{1x} - F_{3x} = ma_x \\ &F_1 \cos 30° - F_{3x} = ma_x \\ &(10.0\,\text{N})(\cos 30°) - 5.0\,\text{N} = (4.0\,\text{kg})a_x \\ &a_x = 0.92\,\text{m/s}^2. \end{aligned}$$

$$\begin{aligned} &\sum F_y = ma_y \\ &F_{1y} + F_{4y} - F_{2y} = ma_y \\ &F_1 \sin 30° + F_{4y} - F_{2y} = ma_y \\ &(10.0\,\text{N})(\sin 30°) + 2.0\,\text{N} - 40.0\,\text{N} = (4.0\,\text{kg})a_y \\ &a_y = -8.3\,\text{m/s}^2. \end{aligned}$$

Thus, the net acceleration is

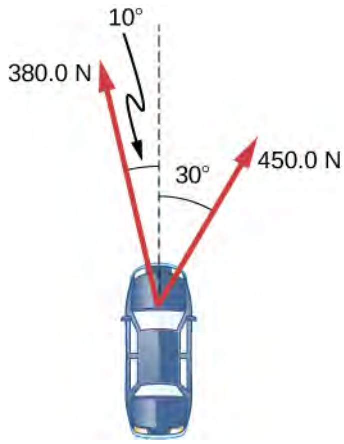
$$\vec{\mathbf{a}} = \left(0.92\,\hat{\mathbf{i}} - 8.3\,\hat{\mathbf{j}}\right)\text{m/s}^2,$$

which is a vector of magnitude $8.4\,\text{m/s}^2$ directed at $276°$ to the positive *x*-axis.

Significance

Numerous examples in everyday life can be found that involve three or more forces acting on a single object, such as cables running from the Golden Gate Bridge or a football player being tackled by three defenders. We can see that the solution of this example is just an extension of what we have already done.

5.5 Check Your Understanding A car has forces acting on it, as shown below. The mass of the car is 1000.0 kg. The road is slick, so friction can be ignored. (a) What is the net force on the car? (b) What is the acceleration of the car?



Newton’s Second Law and Momentum

Newton actually stated his second law in terms of momentum: “The instantaneous rate at which a body’s momentum changes is equal to the net force acting on the body.” (“Instantaneous rate” implies that the derivative is involved.) This can be given by the vector equation

$$\vec{\mathbf{F}}_{\text{net}} = \frac{d\vec{\mathbf{p}}}{dt}. \tag{5.6}$$

This means that Newton’s second law addresses the central question of motion: What causes a change in motion of an object? Momentum was described by Newton as “quantity of motion,” a way of combining both the velocity of an object and its mass. We devote **Linear Momentum and Collisions** to the study of momentum.

For now, it is sufficient to define *momentum* $\vec{\mathbf{p}}$ as the product of the mass of the object m and its velocity $\vec{\mathbf{v}}$:

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}. \tag{5.7}$$

Since velocity is a vector, so is momentum.

It is easy to visualize momentum. A train moving at 10 m/s has more momentum than one that moves at 2 m/s. In everyday life, we speak of one sports team as “having momentum” when they score points against the opposing team.

If we substitute **Equation 5.7** into **Equation 5.6**, we obtain

$$\vec{\mathbf{F}}_{\text{net}} = \frac{d\vec{\mathbf{p}}}{dt} = \frac{d(m\vec{\mathbf{v}})}{dt}.$$

When m is constant, we have

$$\vec{\mathbf{F}}_{\text{net}} = m\frac{d(\vec{\mathbf{v}})}{dt} = m\vec{\mathbf{a}}.$$

Thus, we see that the momentum form of Newton’s second law reduces to the form given earlier in this section.

Explore the **forces at work (https://openstaxcollege.org/l/21forcesatwork)** when **pulling a cart (https://openstaxcollege.org/l/21pullacart)** or pushing a refrigerator, crate, or person. Create an **applied force (https://openstaxcollege.org/l/21forcemotion)** and see how it makes objects move. Put **an object on a ramp (https://openstaxcollege.org/l/21ramp)** and see how it affects its motion.

5.4 | Mass and Weight

Learning Objectives

By the end of the section, you will be able to:

- Explain the difference between mass and weight
- Explain why falling objects on Earth are never truly in free fall
- Describe the concept of weightlessness

Mass and weight are often used interchangeably in everyday conversation. For example, our medical records often show our weight in kilograms but never in the correct units of newtons. In physics, however, there is an important distinction. Weight is the pull of Earth on an object. It depends on the distance from the center of Earth. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon.

Units of Force

The equation $F_{\text{net}} = ma$ is used to define net force in terms of mass, length, and time. As explained earlier, the SI unit of force is the newton. Since $F_{\text{net}} = ma$,

$$1\text{ N} = 1\text{ kg} \cdot \text{m/s}^2.$$

Although almost the entire world uses the newton for the unit of force, in the United States, the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb. Thus, a 225-lb person weighs 1000 N.

Weight and Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law says that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight** $\vec{\mathbf{w}}$, or its force due to gravity acting on an object of mass *m*. Weight can be denoted as a vector because it has a direction; *down* is, by definition, the direction of gravity, and hence, weight is a downward force. The magnitude of weight is denoted as *w*. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration *g*. Using Galileo's result and Newton's second law, we can derive an equation for weight.

Consider an object with mass *m* falling toward Earth. It experiences only the downward force of gravity, which is the weight $\vec{\mathbf{w}}$. Newton's second law says that the magnitude of the net external force on an object is $\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$. We know that the acceleration of an object due to gravity is $\vec{\mathbf{g}}$, or $\vec{\mathbf{a}} = \vec{\mathbf{g}}$. Substituting these into Newton's second law gives us the following equations.

Weight

The gravitational force on a mass is its weight. We can write this in vector form, where $\vec{\mathbf{w}}$ is weight and *m* is mass, as

$$\vec{\mathbf{w}} = m\vec{\mathbf{g}}. \tag{5.8}$$

In scalar form, we can write

$$w = mg. \tag{5.9}$$

Since $g = 9.80\ \text{m/s}^2$ on Earth, the weight of a 1.00-kg object on Earth is 9.80 N:

$$w = mg = (1.00\ \text{kg})(9.80\ \text{m/s}^2) = 9.80\ \text{N}.$$

When the net external force on an object is its weight, we say that it is in **free fall**, that is, the only force acting on the object is gravity. However, when objects on Earth fall downward, they are never truly in free fall because there is always some upward resistance force from the air acting on the object.

Acceleration due to gravity *g* varies slightly over the surface of Earth, so the weight of an object depends on its location and is not an intrinsic property of the object. Weight varies dramatically if we leave Earth's surface. On the Moon, for example, acceleration due to gravity is only $1.67\ \text{m/s}^2$. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that the weight of an object is the gravitational force on it from the nearest large body, such as Earth, the Moon, or the Sun. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of "weightlessness" and "microgravity," they are referring to the phenomenon we call "free fall" in physics. We use the preceding definition of weight, force $\vec{\mathbf{w}}$ due to gravity acting on an object of mass *m*, and we make careful distinctions between free fall and actual weightlessness.

Be aware that weight and mass are different physical quantities, although they are closely related. Mass is an intrinsic property of an object: It is a quantity of matter. The quantity or amount of matter of an object is determined by the numbers of atoms and molecules of various types it contains. Because these numbers do not vary, in Newtonian physics, mass does not vary; therefore, its response to an applied force does not vary. In contrast, weight is the gravitational force acting on an object, so it does vary depending on gravity. For example, a person closer to the center of Earth, at a low elevation such as New Orleans, weighs slightly more than a person who is located in the higher elevation of Denver, even though they may have the same mass.

It is tempting to equate mass to weight, because most of our examples take place on Earth, where the weight of an object varies only a little with the location of the object. In addition, it is difficult to count and identify all of the atoms and molecules in an object, so mass is rarely determined in this manner. If we consider situations in which $\vec{\mathbf{g}}$ is a constant on Earth, we see that weight $\vec{\mathbf{w}}$ is directly proportional to mass *m*, since $\vec{\mathbf{w}} = m\vec{\mathbf{g}}$, that is, the more massive an object is, the more it weighs. Operationally, the masses of objects are determined by comparison with the standard kilogram, as we discussed in **Units and Measurement**. But by comparing an object on Earth with one on the Moon, we can easily see a variation in weight but not in mass. For instance, on Earth, a 5.0-kg object weighs 49 N; on the Moon, where *g* is $1.67\ \text{m/s}^2$, the object weighs 8.4 N. However, the mass of the object is still 5.0 kg on the Moon.

Example 5.8

Clearing a Field

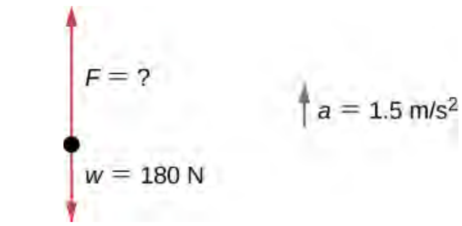
A farmer is lifting some moderately heavy rocks from a field to plant crops. He lifts a stone that weighs 40.0 lb. (about 180 N). What force does he apply if the stone accelerates at a rate of $1.5\ \text{m/s}^2$?

Strategy

We were given the weight of the stone, which we use in finding the net force on the stone. However, we also need to know its mass to apply Newton's second law, so we must apply the equation for weight, $w = mg$, to determine the mass.

Solution

No forces act in the horizontal direction, so we can concentrate on vertical forces, as shown in the following free-body diagram. We label the acceleration to the side; technically, it is not part of the free-body diagram, but it helps to remind us that the object accelerates upward (so the net force is upward).



$$\begin{aligned}
w &= mg \\
m &= \frac{w}{g} = \frac{180 \text{ N}}{9.8 \text{ m/s}^2} = 18 \text{ kg} \\
\sum F &= ma \\
F - w &= ma \\
F - 180 \text{ N} &= (18 \text{ kg})(1.5 \text{ m/s}^2) \\
F - 180 \text{ N} &= 27 \text{ N} \\
F &= 207 \text{ N} = 210 \text{ N to two significant fig es}
\end{aligned}$$

Significance

To apply Newton's second law as the primary equation in solving a problem, we sometimes have to rely on other equations, such as the one for weight or one of the kinematic equations, to complete the solution.

5.6 Check Your Understanding For **Example 5.8**, find the acceleration when the farmer's applied force is 230.0 N.

Can you avoid the boulder field and land safely just before your fuel runs out, as Neil Armstrong did in 1969? This **version of the classic video game (https://openstaxcollege.org/l/21lunarlander)** accurately simulates the real motion of the lunar lander, with the correct mass, thrust, fuel consumption rate, and lunar gravity. The real lunar lander is hard to control.

Use this **interactive simulation (https://openstaxcollege.org/l/21gravityorbits)** to move the Sun, Earth, Moon, and space station to see the effects on their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies, and turn off gravity to see what would happen without it.

5.5 | Newton's Third Law

Learning Objectives

By the end of the section, you will be able to:

- State Newton's third law of motion
- Identify the action and reaction forces in different situations
- Apply Newton's third law to define systems and solve problems of motion

We have thus far considered force as a push or a pull; however, if you think about it, you realize that no push or pull ever occurs by itself. When you push on a wall, the wall pushes back on you. This brings us to **Newton's third law**.

Newton's Third Law of Motion

Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts. Mathematically, if a body *A* exerts a force $\vec{\mathbf{F}}$ on body *B*, then *B* simultaneously exerts a force $-\vec{\mathbf{F}}$ on *A*, or in vector equation form,

$$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}. \tag{5.10}$$

Newton's third law represents a certain symmetry in nature: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as "action-reaction," where the force exerted is the action and the force experienced as a consequence is the reaction. Newton's third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton's third law at work by taking a look at how people move about. Consider a swimmer pushing off the side of a pool (**Figure 5.16**). She pushes against the wall of the pool with her feet and accelerates in the direction opposite that of her push. The wall has exerted an equal and opposite force on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different systems*. In this case, there are two systems that we could investigate: the swimmer and the wall. If we select the swimmer to be the system of interest, as in the figure, then $F_{\text{wall on feet}}$ is an external force on this system and affects its motion. The swimmer moves in the direction of this force. In contrast, the force $F_{\text{feet on wall}}$ acts on the wall, not on our system of interest. Thus, $F_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $F_{\text{wall on feet}}$. The swimmer pushes in the direction opposite that in which she wishes to move. The reaction to her push is thus in the desired direction. In a free-body diagram, such as the one shown in **Figure 5.16**, we never include both forces of an action-reaction pair; in this case, we only use $F_{\text{wall on feet}}$, not $F_{\text{feet on wall}}$.

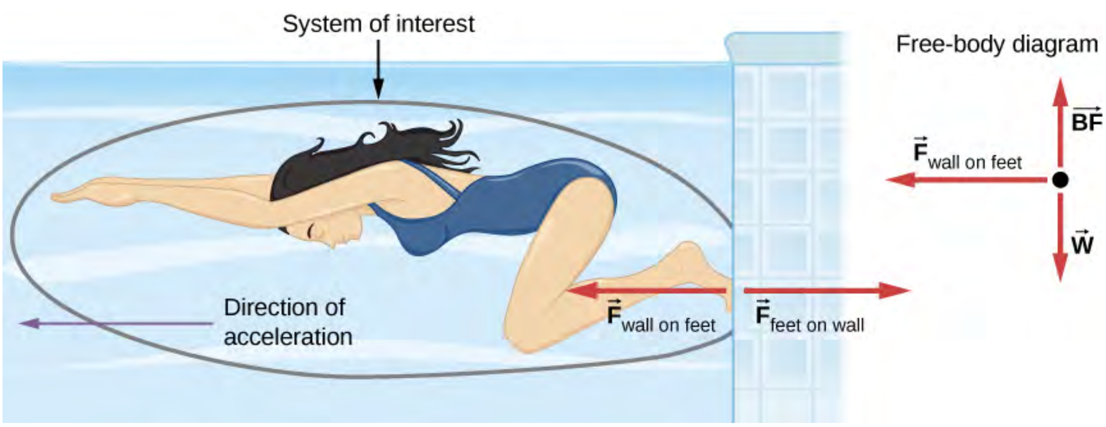


Figure 5.16 When the swimmer exerts a force on the wall, she accelerates in the opposite direction; in other words, the net external force on her is in the direction opposite of $F_{\text{feet on wall}}$. This opposition occurs because, in accordance with Newton's third law, the wall exerts a force $F_{\text{wall on feet}}$ on the swimmer that is equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Thus, the free-body diagram shows only $F_{\text{wall on feet}}$, w (the gravitational force), and *BF*, which is the buoyant force of the water supporting the swimmer's weight. The vertical forces *w* and *BF* cancel because there is no vertical acceleration.

Other examples of Newton's third law are easy to find:

- As a professor paces in front of a whiteboard, he exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes him to accelerate forward.
- A car accelerates forward because the ground pushes forward on the drive wheels, in reaction to the drive wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw the rocks backward.

- Rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber; therefore, the gas exerts a large reaction force forward on the rocket. This reaction force, which pushes a body forward in response to a backward force, is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can more readily expel the exhaust gases.
- Helicopters create lift by pushing air down, thereby experiencing an upward reaction force.
- Birds and airplanes also fly by exerting force on the air in a direction opposite that of whatever force they need. For example, the wings of a bird force air downward and backward to get lift and move forward.
- An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski.
- When a person pulls down on a vertical rope, the rope pulls up on the person (Figure 5.17).

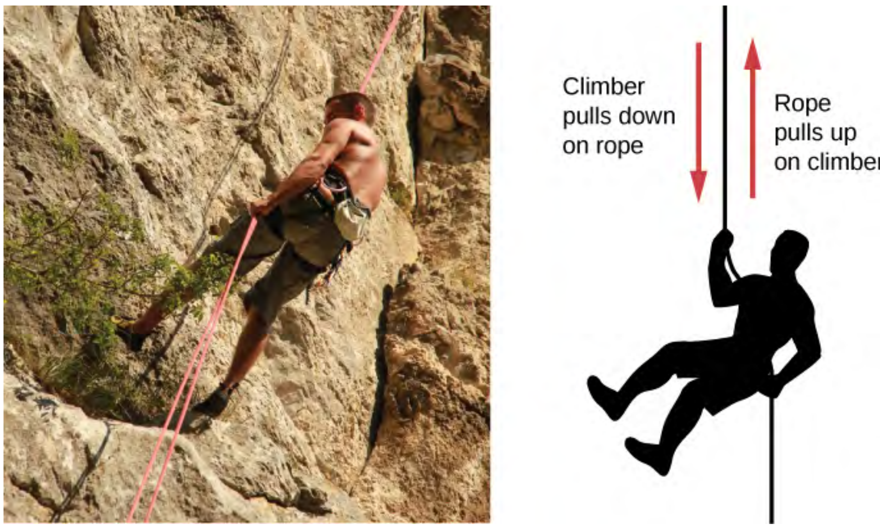


Figure 5.17 When the mountain climber pulls down on the rope, the rope pulls up on the mountain climber.

There are two important features of Newton's third law. First, the forces exerted (the action and reaction) are always equal in magnitude but opposite in direction. Second, these forces are acting on different bodies or systems: *A*'s force acts on *B* and *B*'s force acts on *A*. In other words, the two forces are distinct forces that do not act on the same body. Thus, they do not cancel each other.

For the situation shown in Figure 5.6, the third law indicates that because the chair is pushing upward on the boy with force $\vec{\mathbf{C}}$, he is pushing downward on the chair with force $-\vec{\mathbf{C}}$. Similarly, he is pushing downward with forces $-\vec{\mathbf{F}}$ and $-\vec{\mathbf{T}}$ on the floor and table, respectively. Finally, since Earth pulls downward on the boy with force $\vec{\mathbf{w}}$, he pulls upward on Earth with force $-\vec{\mathbf{w}}$. If that student were to angrily pound the table in frustration, he would quickly learn the painful lesson (avoidable by studying Newton's laws) that the table hits back just as hard.

A person who is walking or running applies Newton's third law instinctively. For example, the runner in Figure 5.18 pushes backward on the ground so that it pushes him forward.

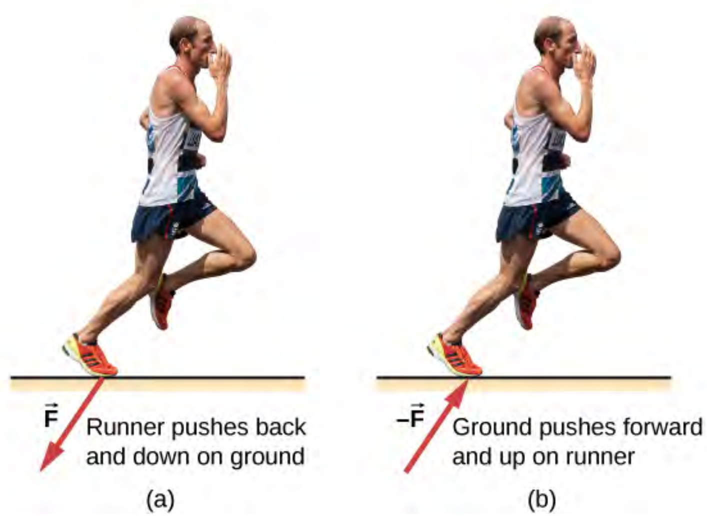


Figure 5.18 The runner experiences Newton’s third law. (a) A force is exerted by the runner on the ground. (b) The reaction force of the ground on the runner pushes him forward.

Example 5.9

Forces on a Stationary Object

The package in **Figure 5.19** is sitting on a scale. The forces on the package are $\vec{\mathbf{S}}$, which is due to the scale, and $-\vec{\mathbf{w}}$, which is due to Earth’s gravitational field. The reaction forces that the package exerts are $-\vec{\mathbf{S}}$ on the scale and $\vec{\mathbf{w}}$ on Earth. Because the package is not accelerating, application of the second law yields

$$\vec{\mathbf{S}} - \vec{\mathbf{w}} = m\vec{\mathbf{a}} = \vec{\mathbf{0}},$$

so

$$\vec{\mathbf{S}} = \vec{\mathbf{w}}.$$

Thus, the scale reading gives the magnitude of the package’s weight. However, the scale does not measure the weight of the package; it measures the force $-\vec{\mathbf{S}}$ on its surface. If the system is accelerating, $\vec{\mathbf{S}}$ and $-\vec{\mathbf{w}}$ would not be equal, as explained in **Applications of Newton’s Laws**.

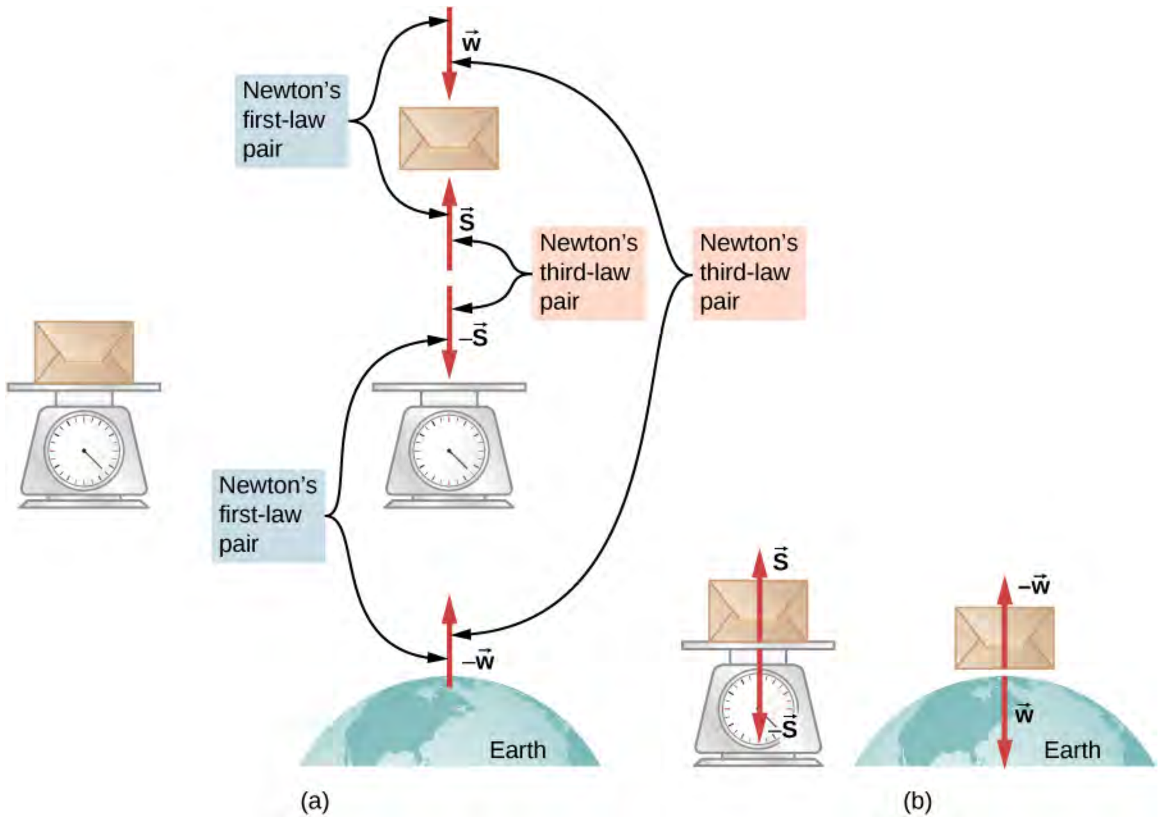


Figure 5.19 (a) The forces on a package sitting on a scale, along with their reaction forces. The force $\vec{\mathbf{w}}$ is the weight of the package (the force due to Earth's gravity) and $\vec{\mathbf{S}}$ is the force of the scale on the package. (b) Isolation of the package-scale system and the package-Earth system makes the action and reaction pairs clear.

Example 5.10

Getting Up to Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall (**Figure 5.20**). Her mass is 65.0 kg, the cart's mass is 12.0 kg, and the equipment's mass is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.

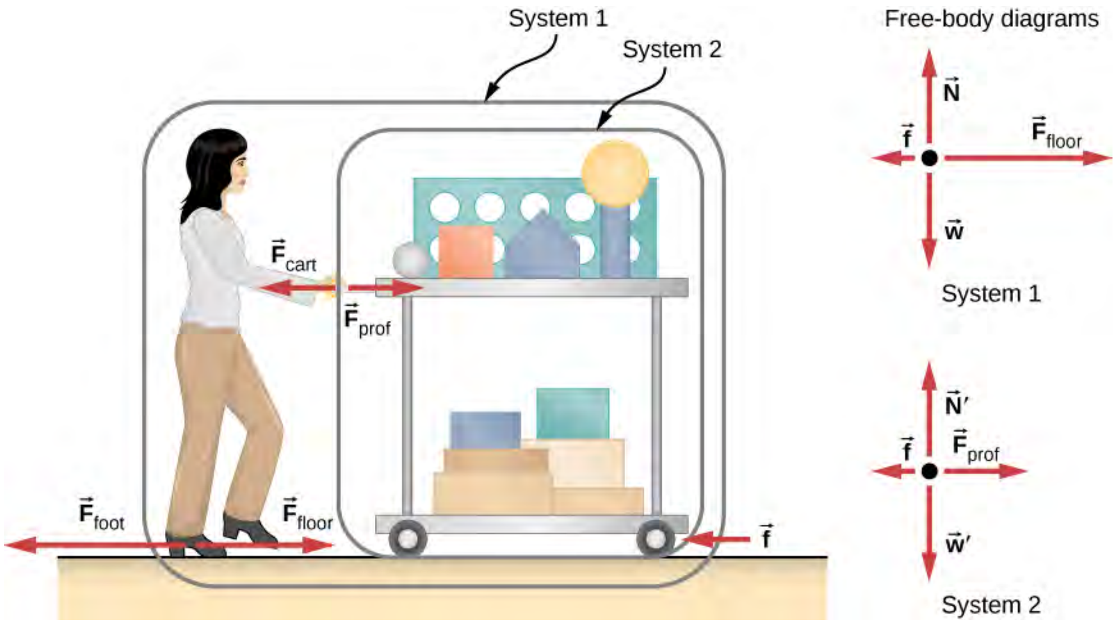


Figure 5.20 A professor pushes the cart with her demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for $\vec{f}$, because it is too small to drawn to scale). System 1 is appropriate for this example, because it asks for the acceleration of the entire group of objects. Only $\vec{F}_{floo}$ and $\vec{f}$ are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is chosen for the next example so that $\vec{F}_{prof}$ is an external force and enters into Newton's second law. The free-body diagrams, which serve as the basis for Newton's second law, vary with the system chosen.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in **Figure 5.20**. The professor pushes backward with a force F_{foot} of 150 N. According to Newton's third law, the floor exerts a forward reaction force F_{floo} of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction. Therefore, the problem is one-dimensional along the horizontal direction. As noted, friction f opposes the motion and is thus in the opposite direction of F_{floo}. We do not include the forces F_{prof} or F_{cart} because these are internal forces, and we do not include F_{foot} because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

$$a = \frac{F_{net}}{m}.$$

The net external force on System 1 is deduced from **Figure 5.20** and the preceding discussion to be

$$F_{net} = F_{floo} - f = 150\,\text{N} - 24.0\,\text{N} = 126\,\text{N}.$$

The mass of System 1 is

$$m = (65.0 + 12.0 + 7.0)\,\text{kg} = 84\,\text{kg}.$$

These values of F_{net} and m produce an acceleration of

$$a = \frac{F_{\text{net}}}{m} = \frac{126\text{ N}}{84\text{ kg}} = 1.5\text{ m/s}^2.$$

Significance

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on the professor. In this case, both forces act on the same system and therefore cancel. Thus, internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example 5.11

Force on the Cart: Choosing a New System

Calculate the force the professor exerts on the cart in **Figure 5.20**, using data from the previous example if needed.

Strategy

If we define the system of interest as the cart plus the equipment (System 2 in **Figure 5.20**), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, F_{prof}, is an external force acting on System 2. F_{prof} was internal to System 1, but it is external to System 2 and thus enters Newton's second law for this system.

Solution

Newton's second law can be used to find F_{prof}. We start with

$$a = \frac{F_{\text{net}}}{m}.$$

The magnitude of the net external force on System 2 is

$$F_{\text{net}} = F_{\text{prof}} - f.$$

We solve for F_{prof}, the desired quantity:

$$F_{\text{prof}} = F_{\text{net}} + f.$$

The value of f is given, so we must calculate net F_{net}. That can be done because both the acceleration and the mass of System 2 are known. Using Newton's second law, we see that

$$F_{\text{net}} = ma,$$

where the mass of System 2 is 19.0 kg ($m = 12.0\text{ kg} + 7.0\text{ kg}$) and its acceleration was found to be $a = 1.5\text{ m/s}^2$ in the previous example. Thus,

$$F_{\text{net}} = ma = (19.0\text{ kg})\left(1.5\text{ m/s}^2\right) = 29\text{ N}.$$

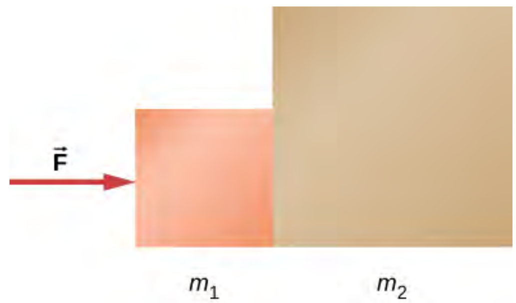
Now we can find the desired force:

$$F_{\text{prof}} = F_{\text{net}} + f = 29\text{ N} + 24.0\text{ N} = 53\text{ N}.$$

Significance

This force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor. The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which are not necessarily the same things).

5.7 Check Your Understanding Two blocks are at rest and in contact on a frictionless surface as shown below, with $m_1 = 2.0\,\text{kg}, \quad m_2 = 6.0\,\text{kg},$ and applied force 24 N. (a) Find the acceleration of the system of blocks. (b) Suppose that the blocks are later separated. What force will give the second block, with the mass of 6.0 kg, the same acceleration as the system of blocks?



View this **video (https://openstaxcollege.org/l/21actionreact)** to watch examples of action and reaction.

View this **video (https://openstaxcollege.org/l/21NewtonsLaws)** to watch examples of Newton's laws and internal and external forces.

5.6 | Common Forces

Learning Objectives

By the end of the section, you will be able to:

- Define normal and tension forces
- Distinguish between real and fictitious forces
- Apply Newton's laws of motion to solve problems involving a variety of forces

Forces are given many names, such as push, pull, thrust, and weight. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. Several of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

A Catalog of Forces: Normal, Tension, and Other Examples of Forces

A catalog of forces will be useful for reference as we solve various problems involving force and motion. These forces include normal force, tension, friction, and spring force.

Normal force

Weight (also called the force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in **Figure 5.21**(a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in **Figure 5.21**(b)? When the bag of dog food is placed on the table, the table sags slightly under the load. This would be noticeable if the load were placed on a card table, but even a sturdy oak table deforms when a force is applied to it. Unless an object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or a trampoline or diving board). The greater the deformation, the greater the restoring force. Thus, when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point, the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly and the sag is slight, so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.

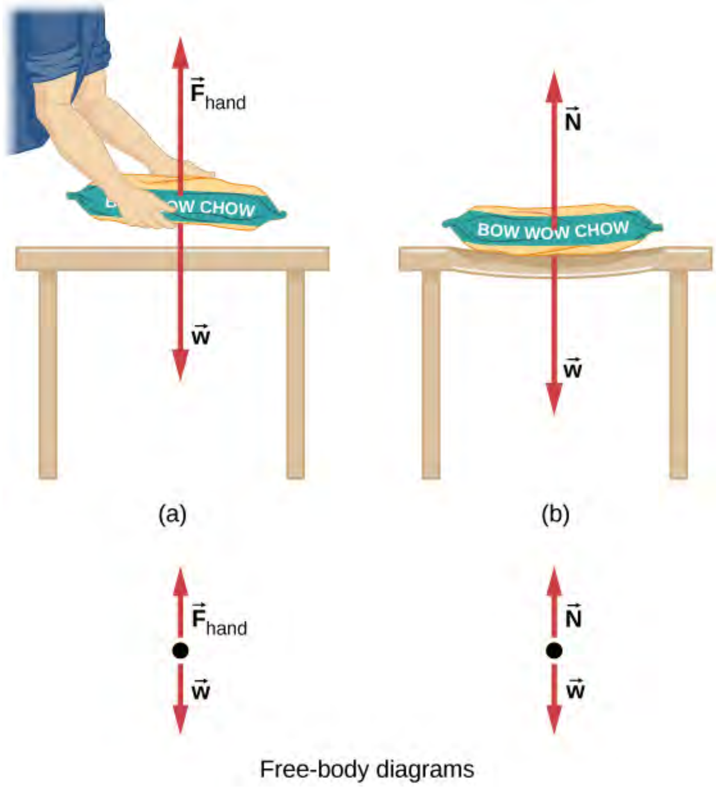


Figure 5.21 (a) The person holding the bag of dog food must supply an upward force $\vec{F}_{hand}$ equal in magnitude and opposite in direction to the weight of the food $\vec{w}$ so that it doesn't drop to the ground. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force $\vec{N}$ equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. If the force supporting the weight of an object, or a load, is perpendicular to the surface of contact between the load and its support, this force is defined as a **normal force** and here is given by the symbol $\vec{N}$. (This is not the newton unit for force, or N.) The word *normal* means perpendicular to a surface. This means that the normal force experienced by an object resting on a horizontal surface can be expressed in vector form as follows:

$$\vec{N} = -m\vec{g}. \tag{5.11}$$

In scalar form, this becomes

$$N = mg. \tag{5.12}$$

The normal force can be less than the object’s weight if the object is on an incline.

Example 5.12

Weight on an Incline

Consider the skier on the slope in Figure 5.22. Her mass including equipment is 60.0 kg. (a) What is her acceleration if friction is negligible? (b) What is her acceleration if friction is 45.0 N?

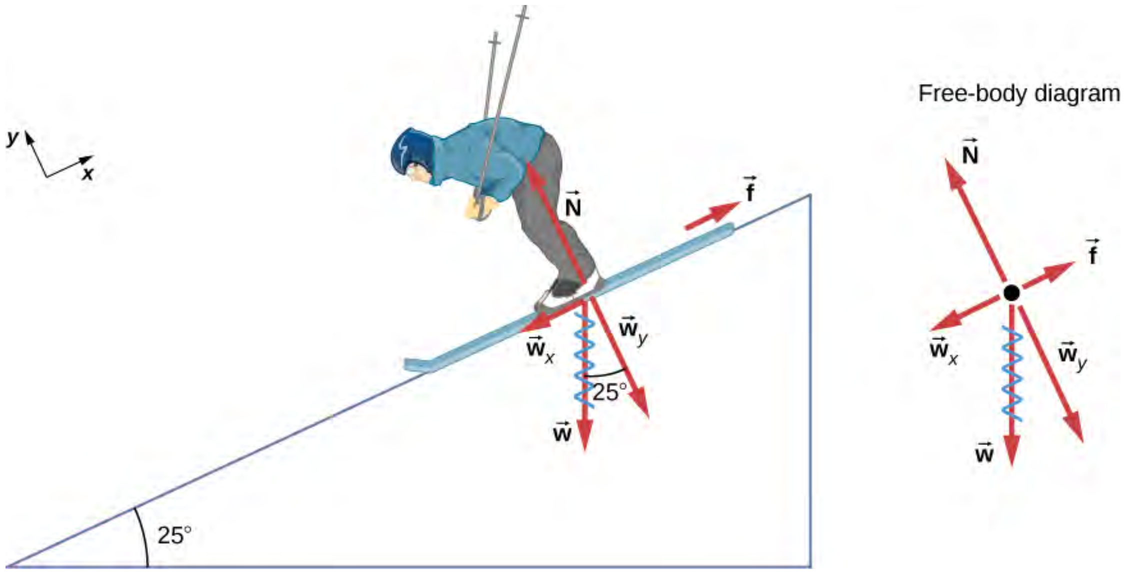


Figure 5.22 Since the acceleration is parallel to the slope and acting down the slope, it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular to it (axes shown to the left of the skier). $\vec{N}$ is perpendicular to the slope and $\vec{f}$ is parallel to the slope, but $\vec{w}$ has components along both axes, namely, w_y and w_x. Here, $\vec{w}$ has a squiggly line to show that it has been replaced by these components. The force $\vec{N}$ is equal in magnitude to w_y, so there is no acceleration perpendicular to the slope, but *f* is less than w_x, so there is a downslope acceleration (along the axis parallel to the slope).

Strategy

This is a two-dimensional problem, since not all forces on the skier (the system of interest) are parallel. The approach we have used in two-dimensional kinematics also works well here. Choose a convenient coordinate system and project the vectors onto its axes, creating two one-dimensional problems to solve. The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. (Motions along mutually perpendicular axes are independent.) We use *x* and *y* for the parallel and perpendicular directions, respectively. This choice of axes simplifies this type of problem, because there is no motion perpendicular to the slope and the acceleration is downslope. Regarding the forces, friction is drawn in opposition to motion (friction always opposes forward motion) and is always parallel to the slope, w_x is drawn parallel to the slope and downslope (it causes the motion of the skier down the slope), and w_y is drawn as the component of weight perpendicular to the slope. Then, we can consider the separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of weight parallel to the slope is

$$w_x = w \sin 25° = mg \sin 25°,$$

and the magnitude of the component of the weight perpendicular to the slope is

$$w_y = w \cos 25° = mg \cos 25°.$$

a. Neglect friction. Since the acceleration is parallel to the slope, we need only consider forces parallel to the slope. (Forces perpendicular to the slope add to zero, since there is no acceleration in that direction.) The forces parallel to the slope are the component of the skier's weight parallel to slope w_x and friction *f*. Using Newton's second law, with subscripts to denote quantities parallel to the slope,

$$a_x = \frac{F_{\text{net } x}}{m}$$

where $F_{\text{net } x} = w_x - mg \sin 25°,$ assuming no friction for this part. Therefore,

$$a_x = \frac{F_{\text{net } x}}{m} = \frac{mg \sin 25°}{m} = g \sin 25°$$

$$\left(9.80 \text{ m/s}^2\right)(0.4226) = 4.14 \text{ m/s}^2$$

is the acceleration.

b. Include friction. We have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is

$$F_{\text{net } x} = w_x - f.$$

Substituting this into Newton's second law, $a_x = F_{\text{net } x}/m,$ gives

$$a_x = \frac{F_{\text{net } x}}{m} = \frac{w_x - f}{m} = \frac{mg \sin 25° - f}{m}.$$

We substitute known values to obtain

$$a_x = \frac{(60.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)(0.4226) - 45.0 \text{ N}}{60.0 \text{ kg}}.$$

This gives us

$$a_x = 3.39 \text{ m/s}^2,$$

which is the acceleration parallel to the incline when there is 45.0 N of opposing friction.

Significance

Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is none. It is a general result that if friction on an incline is negligible, then the acceleration down the incline is $a = g \sin \theta$, regardless of mass. As discussed previously, all objects fall with the same acceleration in the absence of air resistance. Similarly, all objects, regardless of mass, slide down a frictionless incline with the same acceleration (if the angle is the same).

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: a force acting perpendicular to the plane, w_y, and a force acting parallel to the plane, w_x (**Figure 5.23**). The normal force $\vec{\mathbf{N}}$ is typically equal in magnitude and opposite in direction to the perpendicular component of the weight w_y. The force acting parallel to the plane, w_x, causes the object to accelerate down the incline.

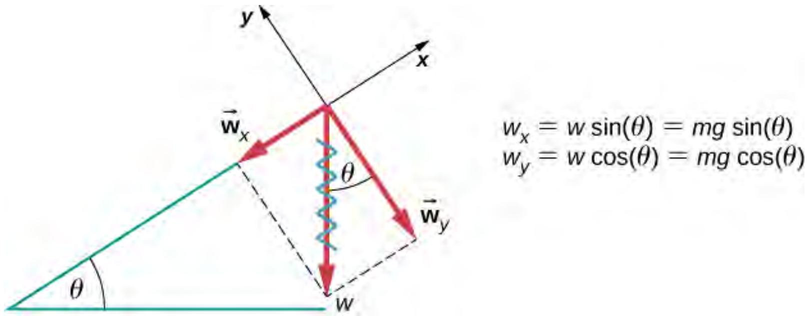


Figure 5.23 An object rests on an incline that makes an angle θ with the horizontal.

Be careful when resolving the weight of the object into components. If the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$w_x = w\sin\theta = mg\sin\theta$$

and

$$w_y = w\cos\theta = mg\cos\theta.$$

We use the second equation to write the normal force experienced by an object resting on an inclined plane:

$$N = mg\cos\theta. \tag{5.13}$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, we draw the right angle formed by the three weight vectors. The angle θ of the incline is the same as the angle formed between w and w_y. Knowing this property, we can use trigonometry to determine the magnitude of the weight components:

$$\cos\theta = \frac{w_y}{w}, \quad w_y = w\cos\theta = mg\sin\theta$$

$$\sin\theta = \frac{w_x}{w}, \quad w_x = w\sin\theta = mg\sin\theta.$$

5.8 **Check Your Understanding** A force of 1150 N acts parallel to a ramp to push a 250-kg gun safe into a moving van. The ramp is frictionless and inclined at $17°$. (a) What is the acceleration of the safe up the ramp? (b) If we consider friction in this problem, with a friction force of 120 N, what is the acceleration of the safe?

Tension

A **tension** is a force along the length of a medium; in particular, it is a pulling force that acts along a stretched flexible connector, such as a rope or cable. The word “tension” comes from a Latin word meaning “to stretch.” Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*.

Any flexible connector, such as a string, rope, chain, wire, or cable, can only exert a pull parallel to its length; thus, a force carried by a flexible connector is a tension with a direction parallel to the connector. Tension is a pull in a connector. Consider the phrase: “You can’t push a rope.” Instead, tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope, as shown in Figure 5.24. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero and the net force is zero. The only external forces acting on the mass are its weight and the tension supplied by the rope. Thus,

$$F_{\text{net}} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight, respectively, and their signs indicate direction, with up being positive. As we proved using Newton’s second law, the tension equals the weight of the supported mass:

$$T = w = mg. \tag{5.14}$$

Thus, for a 5.00-kg mass (neglecting the mass of the rope), we see that

$$T = mg = (5.00\,\text{kg})\left(9.80\,\text{m/s}^2\right) = 49.0\,\text{N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

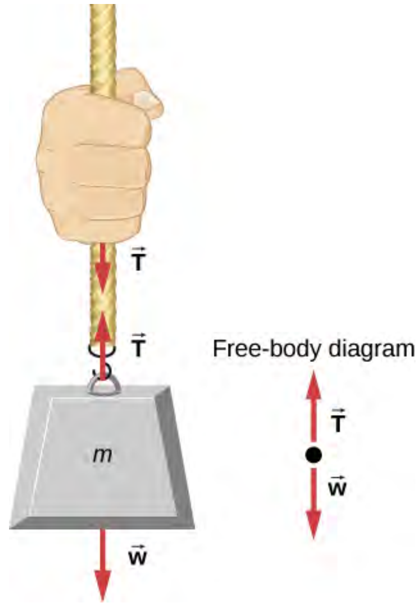


Figure 5.24 When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\vec{\mathbf{T}}$, that force must be parallel to the length of the rope, as shown. By Newton's third law, the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). The rope is the medium that carries the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a tendon, or a bicycle brake cable. If there is no friction, the tension transmission is undiminished; only its direction changes, and it is always parallel to the flexible connector, as shown in **Figure 5.25**.

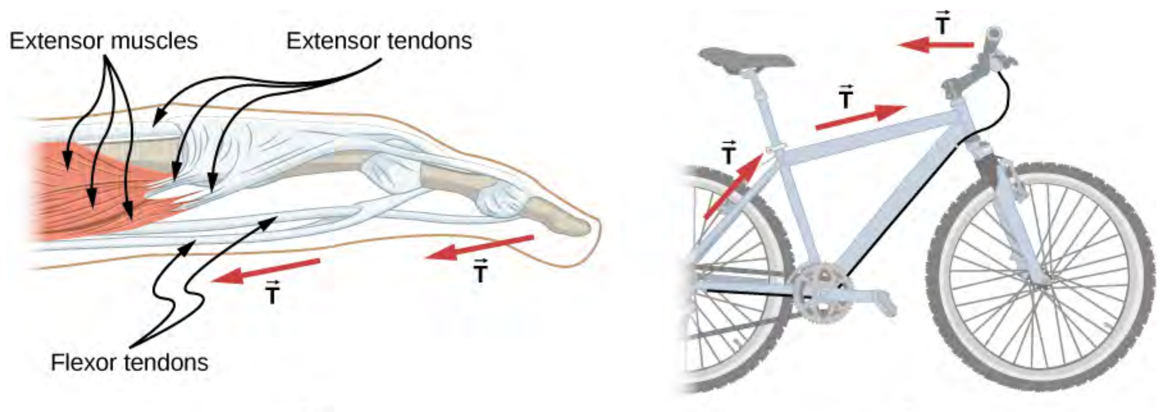


Figure 5.25 (a) Tendons in the finger carry force *T* from the muscles to other parts of the finger, usually changing the force's direction but not its magnitude (the tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension *T* from the brake lever on the handlebars to the brake mechanism. Again, the direction but not the magnitude of *T* is changed.

Example 5.13

What Is the Tension in a Tightrope?

Calculate the tension in the wire supporting the 70.0-kg tightrope walker shown in **Figure 5.26**.

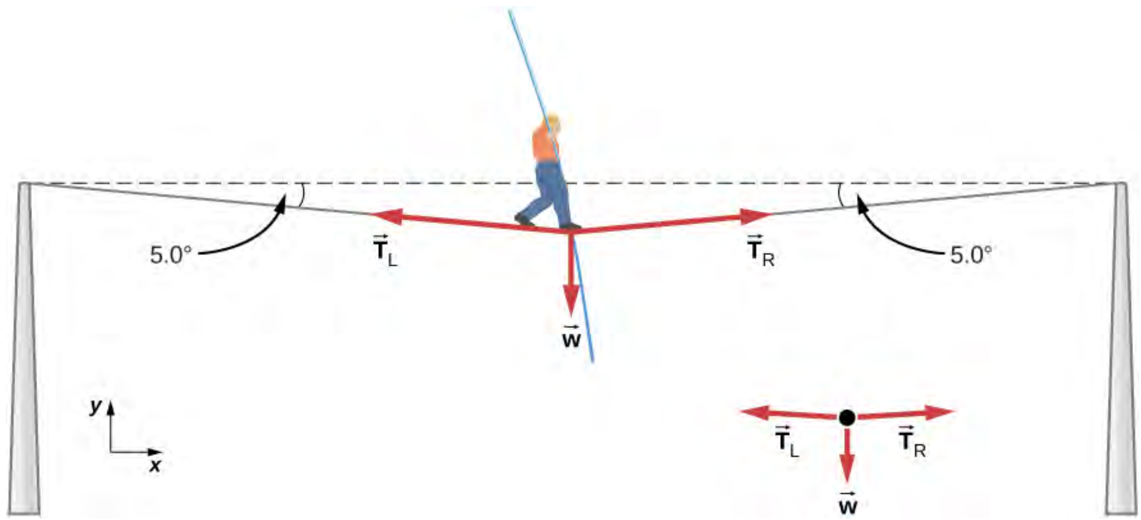


Figure 5.26 The weight of a tightrope walker causes a wire to sag by $5.0°$. The system of interest is the point in the wire at which the tightrope walker is standing.

Strategy

As you can see in **Figure 5.26**, the wire is bent under the person's weight. Thus, the tension on either side of the person has an upward component that can support his weight. As usual, forces are vectors represented pictorially by arrows that have the same direction as the forces and lengths proportional to their magnitudes. The system is the tightrope walker, and the only external forces acting on him are his weight $\vec{w}$ and the two tensions $\vec{T}_L$ (left tension) and $\vec{T}_R$ (right tension). It is reasonable to neglect the weight of the wire. The net external force is zero, because the system is static. We can use trigonometry to find the tensions. One conclusion is possible at the outset—we can see from **Figure 5.26**(b) that the magnitudes of the tensions T_L and T_R must be equal. We know this because there is no horizontal acceleration in the rope and the only forces acting to the left and right are

T_L and T_R . Thus, the magnitude of those horizontal components of the forces must be equal so that they cancel each other out.

Whenever we have two-dimensional vector problems in which no two vectors are parallel, the easiest method of solution is to pick a convenient coordinate system and project the vectors onto its axes. In this case, the best coordinate system has one horizontal axis (*x*) and one vertical axis (*y*).

Solution

First, we need to resolve the tension vectors into their horizontal and vertical components. It helps to look at a new free-body diagram showing all horizontal and vertical components of each force acting on the system (**Figure 5.27**).

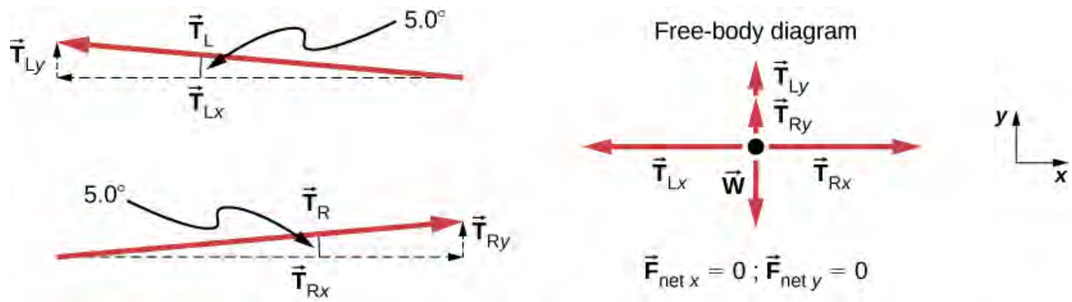


Figure 5.27 When the vectors are projected onto vertical and horizontal axes, their components along these axes must add to zero, since the tightrope walker is stationary. The small angle results in *T* being much greater than *w*.

Consider the horizontal components of the forces (denoted with a subscript *x*):

$$F_{\text{net}\,x} = T_{Rx} - T_{Lx}.$$

The net external horizontal force $F_{\text{net}\,x} = 0,$ since the person is stationary. Thus,

$$\begin{aligned} F_{\text{net}\,x} &= 0 = T_{Rx} - T_{Lx} \\ T_{Lx} &= T_{Rx}. \end{aligned}$$

Now observe **Figure 5.27**. You can use trigonometry to determine the magnitude of T_L and T_R :

$$\begin{aligned} \cos 5.0° &= \frac{T_{Lx}}{T_L}, \quad T_{Lx} = T_L \cos 5.0° \\ \cos 5.0° &= \frac{T_{Rx}}{T_R}, \quad T_{Rx} = T_R \cos 5.0°. \end{aligned}$$

Equating T_{Lx} and T_{Rx}:

$$T_L \cos 5.0° = T_R \cos 5.0°.$$

Thus,

$$T_L = T_R = T,$$

as predicted. Now, considering the vertical components (denoted by a subscript *y*), we can solve for *T*. Again, since the person is stationary, Newton's second law implies that $F_{\text{net}\,y} = 0$. Thus, as illustrated in the free-body diagram,

$$F_{\text{net}\,y} = T_{Ly} + T_{Ry} - w = 0.$$

We can use trigonometry to determine the relationships among $T_{Ly}, T_{Ry},$ and *T*. As we determined from the analysis in the horizontal direction, $T_L = T_R = T$:

$$\begin{aligned}\sin 5.0° &= \frac{T_{Ly}}{T_L}, \quad T_{Ly} = T_L \sin 5.0° = T \sin 5.0° \\ \sin 5.0° &= \frac{T_{Ry}}{T_R}, \quad T_{Ry} = T_R \sin 5.0° = T \sin 5.0°.\end{aligned}$$

Now we can substitute the vales for T_{Ly} and T_{Ry} , into the net force equation in the vertical direction:

$$\begin{aligned}F_{\text{net } y} &= T_{Ly} + T_{Ry} - w = 0 \\ F_{\text{net } y} &= T \sin 5.0° + T \sin 5.0° - w = 0 \\ 2T \sin 5.0° - w &= 0 \\ 2T \sin 5.0° &= w\end{aligned}$$

and

$$T = \frac{w}{2 \sin 5.0°} = \frac{mg}{2 \sin 5.0°},$$

so

$$T = \frac{(70.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)}{2(0.0872)},$$

and the tension is

$$T = 3930 \text{ N}.$$

Significance

The vertical tension in the wire acts as a force that supports the weight of the tightrope walker. The tension is almost six times the 686-N weight of the tightrope walker. Since the wire is nearly horizontal, the vertical component of its tension is only a fraction of the tension in the wire. The large horizontal components are in opposite directions and cancel, so most of the tension in the wire is not used to support the weight of the tightrope walker.

If we wish to create a large tension, all we have to do is exert a force perpendicular to a taut flexible connector, as illustrated in **Figure 5.26**. As we saw in **Example 5.13**, the weight of the tightrope walker acts as a force perpendicular to the rope. We saw that the tension in the rope is related to the weight of the tightrope walker in the following way:

$$T = \frac{w}{2 \sin \theta}.$$

We can extend this expression to describe the tension *T* created when a perpendicular force $(F_\perp)$ is exerted at the middle of a flexible connector:

$$T = \frac{F_\perp}{2 \sin \theta}.$$

The angle between the horizontal and the bent connector is represented by θ . In this case, *T* becomes large as θ approaches zero. Even the relatively small weight of any flexible connector will cause it to sag, since an infinite tension would result if it were horizontal (i.e., $\theta = 0$ and sin $\theta = 0$). For example, **Figure 5.28** shows a situation where we wish to pull a car out of the mud when no tow truck is available. Each time the car moves forward, the chain is tightened to keep it as straight as possible. The tension in the chain is given by $T = \frac{F_\perp}{2 \sin \theta},$ and since θ is small, *T* is large. This situation is analogous to the tightrope walker, except that the tensions shown here are those transmitted to the car and the tree rather than those acting at the point where $F_\perp$ is applied.

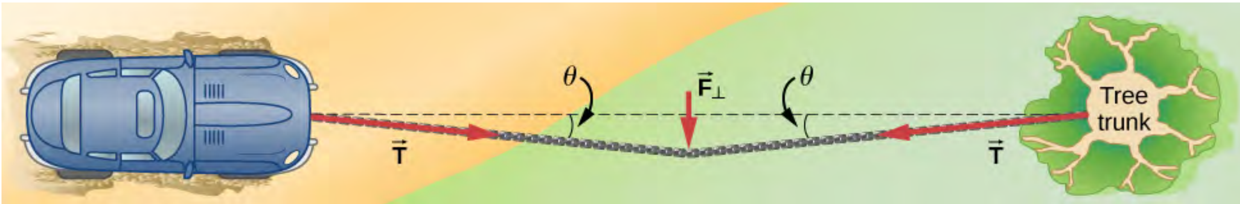


Figure 5.28 We can create a large tension in the chain—and potentially a big mess—by pushing on it perpendicular to its length, as shown.

5.9 **Check Your Understanding** One end of a 3.0-m rope is tied to a tree; the other end is tied to a car stuck in the mud. The motorist pulls sideways on the midpoint of the rope, displacing it a distance of 0.25 m. If he exerts a force of 200.0 N under these conditions, determine the force exerted on the car.

In **Applications of Newton's Laws**, we extend the discussion on tension in a cable to include cases in which the angles shown are not equal.

Friction

Friction is a resistive force opposing motion or its tendency. Imagine an object at rest on a horizontal surface. The net force acting on the object must be zero, leading to equality of the weight and the normal force, which act in opposite directions. If the surface is tilted, the normal force balances the component of the weight perpendicular to the surface. If the object does not slide downward, the component of the weight parallel to the inclined plane is balanced by friction. Friction is discussed in greater detail in the next chapter.

Spring force

A spring is a special medium with a specific atomic structure that has the ability to restore its shape, if deformed. To restore its shape, a spring exerts a restoring force that is proportional to and in the opposite direction in which it is stretched or compressed. This is the statement of a law known as Hooke's law, which has the mathematical form

$$\vec{\mathbf{F}} = -k\vec{\mathbf{x}}.$$

The constant of proportionality k is a measure of the spring's stiffness. The line of action of this force is parallel to the spring axis, and the sense of the force is in the opposite direction of the displacement vector (**Figure 5.29**). The displacement must be measured from the relaxed position; $x = 0$ when the spring is relaxed.

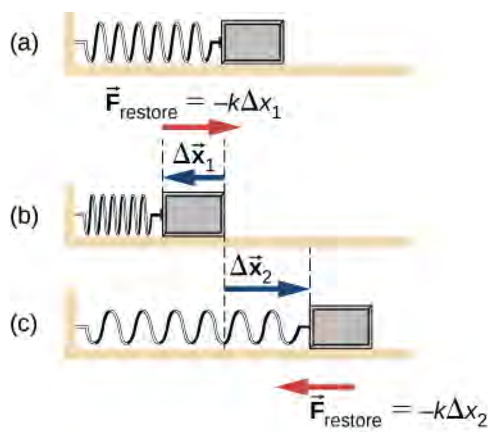


Figure 5.29 A spring exerts its force proportional to a displacement, whether it is compressed or stretched. (a) The spring is in a relaxed position and exerts no force on the block. (b) The spring is compressed by displacement $\Delta\vec{\mathbf{x}}_1$ of the object and exerts restoring force $-k\Delta\vec{\mathbf{x}}_1$. (c) The spring is stretched by displacement $\Delta\vec{\mathbf{x}}_2$ of the object and exerts restoring force $-k\Delta\vec{\mathbf{x}}_2$.

Real Forces and Inertial Frames

There is another distinction among forces: Some forces are real, whereas others are not. *Real forces* have some physical origin, such as a gravitational pull. In contrast, *fictitious forces* arise simply because an observer is in an accelerating or noninertial frame of reference, such as one that rotates (like a merry-go-round) or undergoes linear acceleration (like a car slowing down). For example, if a satellite is heading due north above Earth's Northern Hemisphere, then to an observer on Earth, it will appear to experience a force to the west that has no physical origin. Instead, Earth is rotating toward the east and moves east under the satellite. In Earth's frame, this looks like a westward force on the satellite, or it can be interpreted as a violation of Newton's first law (the law of inertia). We can identify a fictitious force by asking the question, "What is the reaction force?" If we cannot name the reaction force, then the force we are considering is fictitious. In the example of the satellite, the reaction force would have to be an eastward force on Earth. Recall that an inertial frame of reference is one in which all forces are real and, equivalently, one in which Newton's laws have the simple forms given in this chapter.

Earth's rotation is slow enough that Earth is nearly an inertial frame. You ordinarily must perform precise experiments to observe fictitious forces and the slight departures from Newton's laws, such as the effect just described. On a large scale, such as for the rotation of weather systems and ocean currents, the effects can be easily observed (**Figure 5.30**).

Figure 5.30 Hurricane Fran is shown heading toward the southeastern coast of the United States in September 1996. Notice the characteristic "eye" shape of the hurricane. This is a result of the Coriolis effect, which is the deflection of objects (in this case, air) when considered in a rotating frame of reference, like the spin of Earth.

The crucial factor in determining whether a frame of reference is inertial is whether it accelerates or rotates relative to a known inertial frame. Unless stated otherwise, all phenomena discussed in this text are in inertial frames.

The forces discussed in this section are real forces, but they are not the only real forces. Lift and thrust, for example, are more specialized real forces. In the long list of forces, are some more basic than others? Are some different manifestations of the same underlying force? The answer to both questions is yes, as you will see in the treatment of modern physics later in the text.

Explore forces and motion in this **interactive simulation (https://openstaxcollege.org/l/21ramp)** as you push household objects up and down a ramp. Lower and raise the ramp to see how the angle of inclination affects the parallel forces. Graphs show forces, energy, and work.

Stretch and compress springs in this **activity (https://openstaxcollege.org/l/21hookeslaw)** to explore the relationships among force, spring constant, and displacement. Investigate what happens when two springs are connected in series and in parallel.

5.7 | Drawing Free-Body Diagrams

Learning Objectives

By the end of the section, you will be able to:

- Explain the rules for drawing a free-body diagram
- Construct free-body diagrams for different situations

The first step in describing and analyzing most phenomena in physics involves the careful drawing of a free-body diagram. Free-body diagrams have been used in examples throughout this chapter. Remember that a free-body diagram must only include the external forces acting on the body of interest. Once we have drawn an accurate free-body diagram, we can apply Newton's first law if the body is in equilibrium (balanced forces; that is, $F_{\text{net}} = 0$) or Newton's second law if the body is accelerating (unbalanced force; that is, $F_{\text{net}} \neq 0$).

In **Forces**, we gave a brief problem-solving strategy to help you understand free-body diagrams. Here, we add some details to the strategy that will help you in constructing these diagrams.

Problem-Solving Strategy: Constructing Free-Body Diagrams

Observe the following rules when constructing a free-body diagram:

1. Draw the object under consideration; it does not have to be artistic. At first, you may want to draw a circle around the object of interest to be sure you focus on labeling the forces acting on the object. If you are treating the object as a particle (no size or shape and no rotation), represent the object as a point. We often place this point at the origin of an *xy*-coordinate system.
2. Include all forces that act on the object, representing these forces as vectors. Consider the types of forces described in **Common Forces**—normal force, friction, tension, and spring force—as well as weight and applied force. Do not include the net force on the object. With the exception of gravity, all of the forces we have discussed require direct contact with the object. However, forces that the object exerts on its environment must not be included. We never include both forces of an action-reaction pair.
3. Convert the free-body diagram into a more detailed diagram showing the *x*- and *y*-components of a given force (this is often helpful when solving a problem using Newton's first or second law). In this case, place a squiggly line through the original vector to show that it is no longer in play—it has been replaced by its *x*- and *y*-components.
4. If there are two or more objects, or bodies, in the problem, draw a separate free-body diagram for each object.

Note: If there is acceleration, we do not directly include it in the free-body diagram; however, it may help to indicate acceleration outside the free-body diagram. You can label it in a different color to indicate that it is separate from the free-body diagram.

Let's apply the problem-solving strategy in drawing a free-body diagram for a sled. In **Figure 5.31**(a), a sled is pulled by force **P** at an angle of $30°$. In part (b), we show a free-body diagram for this situation, as described by steps 1 and 2 of the problem-solving strategy. In part (c), we show all forces in terms of their *x*- and *y*-components, in keeping with step 3.

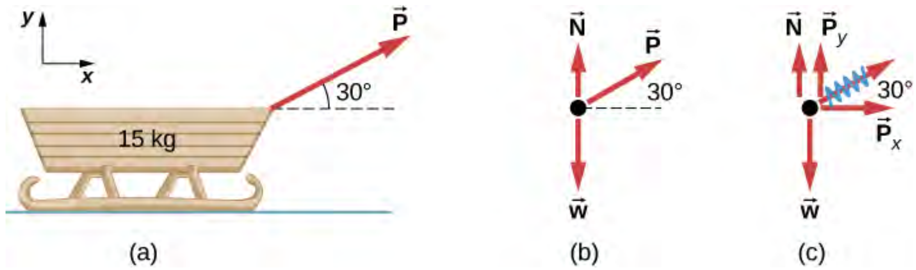


Figure 5.31 (a) A moving sled is shown as (b) a free-body diagram and (c) a free-body diagram with force components.

Example 5.14

Two Blocks on an Inclined Plane

Construct the free-body diagram for object A and object B in **Figure 5.32**.

Strategy

We follow the four steps listed in the problem-solving strategy.

Solution

We start by creating a diagram for the first object of interest. In **Figure 5.32**(a), object A is isolated (circled) and represented by a dot.

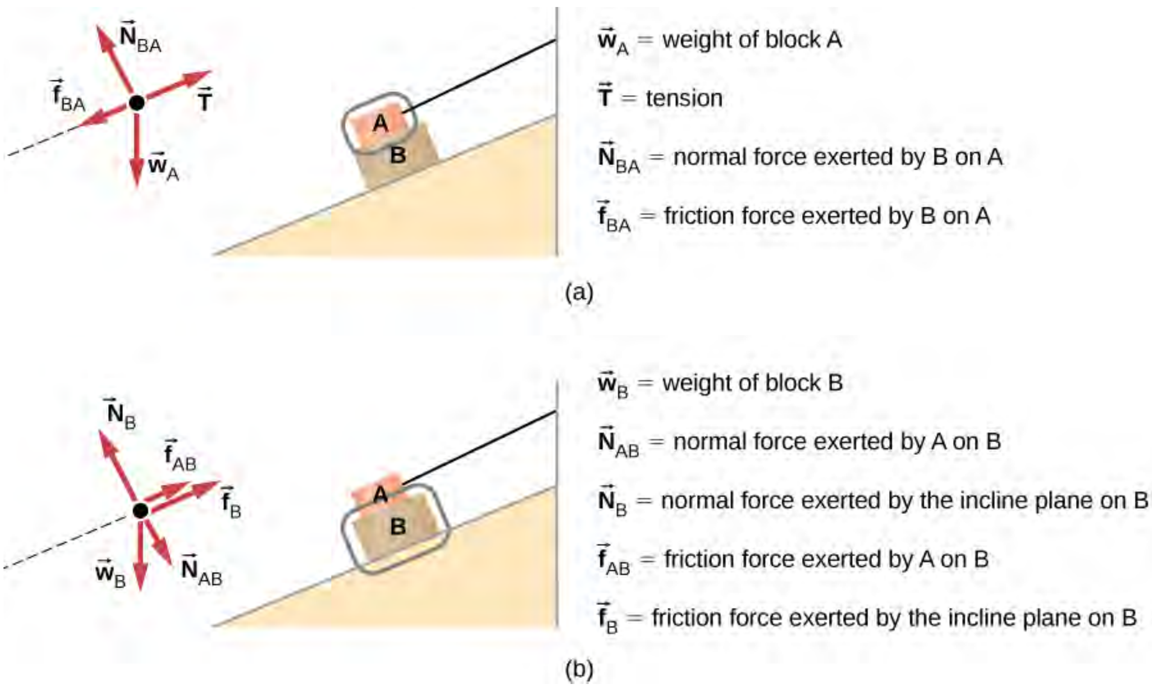


Figure 5.32 (a) The free-body diagram for isolated object A. (b) The free-body diagram for isolated object B. Comparing the two drawings, we see that friction acts in the opposite direction in the two figures. Because object A experiences a force that tends to pull it to the right, friction must act to the left. Because object B experiences a component of its weight that pulls it to the left, down the incline, the friction force must oppose it and act up the ramp. Friction always acts opposite the intended direction of motion.

We now include any force that acts on the body. Here, no applied force is present. The weight of the object acts as a force pointing vertically downward, and the presence of the cord indicates a force of tension pointing away from the object. Object A has one interface and hence experiences a normal force, directed away from the interface. The source of this force is object B, and this normal force is labeled accordingly. Since object B has a tendency to slide down, object A has a tendency to slide up with respect to the interface, so the friction f_{BA} is directed downward parallel to the inclined plane.

As noted in step 4 of the problem-solving strategy, we then construct the free-body diagram in **Figure 5.32**(b) using the same approach. Object B experiences two normal forces and two friction forces due to the presence of two contact surfaces. The interface with the inclined plane exerts external forces of N_B and f_B, and the interface with object B exerts the normal force N_{AB} and friction f_{AB}; N_{AB} is directed away from object B, and f_{AB} is opposing the tendency of the relative motion of object B with respect to object A.

Significance

The object under consideration in each part of this problem was circled in gray. When you are first learning how to draw free-body diagrams, you will find it helpful to circle the object before deciding what forces are acting on that particular object. This focuses your attention, preventing you from considering forces that are not acting on the body.

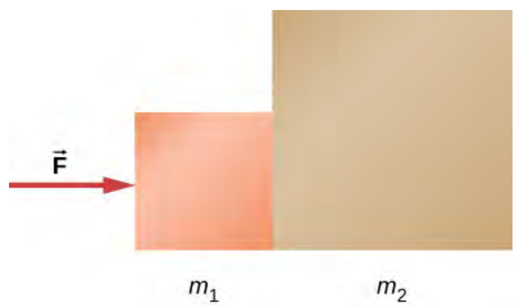
Example 5.15

Two Blocks in Contact

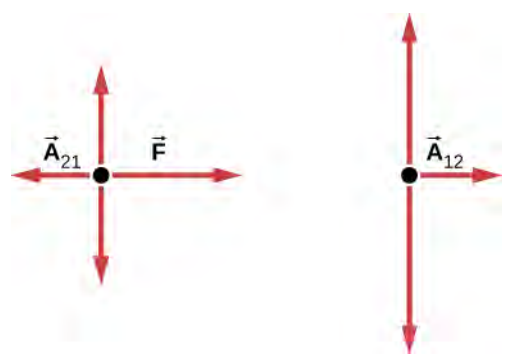
A force is applied to two blocks in contact, as shown.

Strategy

Draw a free-body diagram for each block. Be sure to consider Newton’s third law at the interface where the two blocks touch.



Solution



Significance

$\vec{A}_{21}$ is the action force of block 2 on block 1. $\vec{A}_{12}$ is the reaction force of block 1 on block 2. We use these free-body diagrams in **Applications of Newton’s Laws**.

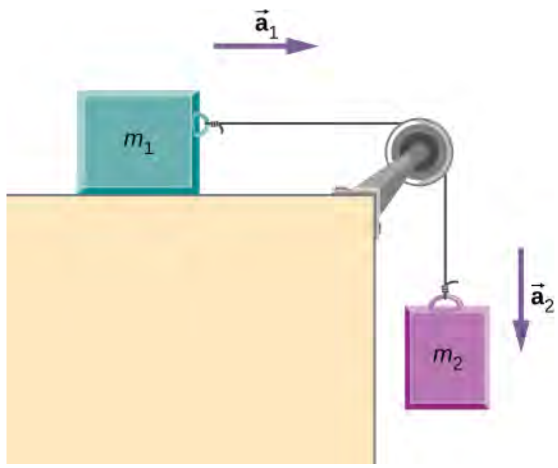
Example 5.16

Block on the Table (Coupled Blocks)

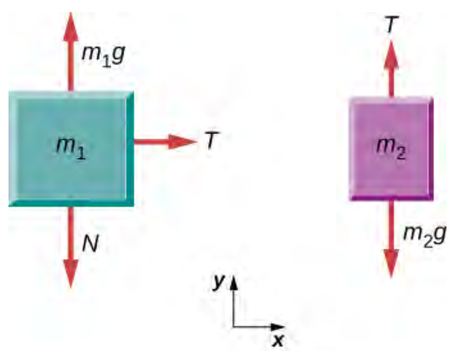
A block rests on the table, as shown. A light rope is attached to it and runs over a pulley. The other end of the rope is attached to a second block. The two blocks are said to be coupled. Block m_2 exerts a force due to its weight, which causes the system (two blocks and a string) to accelerate.

Strategy

We assume that the string has no mass so that we do not have to consider it as a separate object. Draw a free-body diagram for each block.



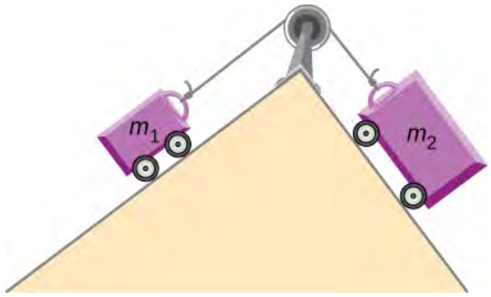
Solution



Significance

Each block accelerates (notice the labels shown for $\vec{a}_1$ and $\vec{a}_2$); however, assuming the string remains taut, they accelerate at the same rate. Thus, we have $\vec{a}_1 = \vec{a}_2$. If we were to continue solving the problem, we could simply call the acceleration $\vec{a}$. Also, we use two free-body diagrams because we are usually finding tension *T*, which may require us to use a system of two equations in this type of problem. The tension is the same on both m_1 and m_2.

5.10 Check Your Understanding (a) Draw the free-body diagram for the situation shown. (b) Redraw it showing components; use *x*-axes parallel to the two ramps.



View this **simulation (https://openstaxcollege.org/l/21forcemotion)** to predict, qualitatively, how an external force will affect the speed and direction of an object's motion. Explain the effects with the help of a free-body diagram. Use free-body diagrams to draw position, velocity, acceleration, and force graphs, and vice versa. Explain how the graphs relate to one another. Given a scenario or a graph, sketch all four graphs.

CHAPTER 5 REVIEW

KEY TERMS

dynamics study of how forces affect the motion of objects and systems

external force force acting on an object or system that originates outside of the object or system

force push or pull on an object with a specific magnitude and direction; can be represented by vectors or expressed as a multiple of a standard force

free fall situation in which the only force acting on an object is gravity

free-body diagram sketch showing all external forces acting on an object or system; the system is represented by a single isolated point, and the forces are represented by vectors extending outward from that point

Hooke's law in a spring, a restoring force proportional to and in the opposite direction of the imposed displacement

inertia ability of an object to resist changes in its motion

inertial reference frame reference frame moving at constant velocity relative to an inertial frame is also inertial; a reference frame accelerating relative to an inertial frame is not inertial

law of inertia see Newton's first law of motion

net external force vector sum of all external forces acting on an object or system; causes a mass to accelerate

newton SI unit of force; 1 N is the force needed to accelerate an object with a mass of 1 kg at a rate of $1\ \text{m/s}^2$

Newton's first law of motion body at rest remains at rest or, if in motion, remains in motion at constant velocity unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system and is inversely proportional to its mass

Newton's third law of motion whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts

normal force force supporting the weight of an object, or a load, that is perpendicular to the surface of contact between the load and its support; the surface applies this force to an object to support the weight of the object

tension pulling force that acts along a stretched flexible connector, such as a rope or cable

thrust reaction force that pushes a body forward in response to a backward force

weight force $\vec{\mathbf{w}}$ due to gravity acting on an object of mass m

KEY EQUATIONS

Net external force	$\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots$
Newton's first law	$\vec{\mathbf{v}} = \text{constant when } \vec{\mathbf{F}}_{\text{net}} = \vec{\mathbf{0}}\ \text{N}$
Newton's second law, vector form	$\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$
Newton's second law, scalar form	$F_{\text{net}} = ma$
Newton's second law, component form	$\sum \vec{\mathbf{F}}_x = m\vec{\mathbf{a}}_x, \sum \vec{\mathbf{F}}_y = m\vec{\mathbf{a}}_y, \text{ and } \sum \vec{\mathbf{F}}_z = m\vec{\mathbf{a}}_z.$
Newton's second law, momentum form	$\vec{\mathbf{F}}_{\text{net}} = \dfrac{d\vec{\mathbf{p}}}{dt}$
Definition of weight, vector form	$\vec{\mathbf{w}} = m\vec{\mathbf{g}}$

Definition of weight, scalar form	$w = mg$
Newton’s third law	$\vec{\mathbf{F}}_{AB} = -\vec{\mathbf{F}}_{BA}$
Normal force on an object resting on a horizontal surface, vector form	$\vec{\mathbf{N}} = -m\vec{\mathbf{g}}$
Normal force on an object resting on a horizontal surface, scalar form	$N = mg$
Normal force on an object resting on an inclined plane, scalar form	$N = mg\cos\theta$
Tension in a cable supporting an object of mass *m* at rest, scalar form	$T = w = mg$

SUMMARY

5.1 Forces

- Dynamics is the study of how forces affect the motion of objects, whereas kinematics simply describes the way objects move.
- Force is a push or pull that can be defined in terms of various standards, and it is a vector that has both magnitude and direction.
- External forces are any outside forces that act on a body. A free-body diagram is a drawing of all external forces acting on a body.
- The SI unit of force is the newton (N).

5.2 Newton's First Law

- According to Newton’s first law, there must be a cause for any change in velocity (a change in either magnitude or direction) to occur. This law is also known as the law of inertia.
- Friction is an external force that causes an object to slow down.
- Inertia is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object’s mass.
- If an object’s velocity relative to a given frame is constant, then the frame is inertial. This means that for an inertial reference frame, Newton’s first law is valid.
- Equilibrium is achieved when the forces on a system are balanced.
- A net force of zero means that an object is either at rest or moving with constant velocity; that is, it is not accelerating.

5.3 Newton's Second Law

- An external force acts on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton’s second law of motion says that the net external force on an object with a certain mass is directly proportional to and in the same direction as the acceleration of the object.
- Newton’s second law can also describe net force as the instantaneous rate of change of momentum. Thus, a net external force causes nonzero acceleration.

5.4 Mass and Weight

- Mass is the quantity of matter in a substance.
- The weight of an object is the net force on a falling object, or its gravitational force. The object experiences acceleration due to gravity.
- Some upward resistance force from the air acts on all falling objects on Earth, so they can never truly be in free fall.

- Careful distinctions must be made between free fall and weightlessness using the definition of weight as force due to gravity acting on an object of a certain mass.

5.5 Newton's Third Law

- Newton's third law of motion represents a basic symmetry in nature, with an experienced force equal in magnitude and opposite in direction to an exerted force.
- Two equal and opposite forces do not cancel because they act on different systems.
- Action-reaction pairs include a swimmer pushing off a wall, helicopters creating lift by pushing air down, and an octopus propelling itself forward by ejecting water from its body. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.
- Choosing a system is an important analytical step in understanding the physics of a problem and solving it.

5.6 Common Forces

- When an object rests on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force.
- When an object rests on a nonaccelerating horizontal surface, the magnitude of the normal force is equal to the weight of the object.
- When an object rests on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be resolved into components that act perpendicular and parallel to the surface of the plane.
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension. When a rope supports the weight of an object at rest, the tension in the rope is equal to the weight of the object. If the object is accelerating, tension is greater than weight, and if it is decelerating, tension is less than weight.
- The force of friction is a force experienced by a moving object (or an object that has a tendency to move) parallel to the interface opposing the motion (or its tendency).
- The force developed in a spring obeys Hooke's law, according to which its magnitude is proportional to the displacement and has a sense in the opposite direction of the displacement.
- Real forces have a physical origin, whereas fictitious forces occur because the observer is in an accelerating or noninertial frame of reference.

5.7 Drawing Free-Body Diagrams

- To draw a free-body diagram, we draw the object of interest, draw all forces acting on that object, and resolve all force vectors into *x*- and *y*-components. We must draw a separate free-body diagram for each object in the problem.
- A free-body diagram is a useful means of describing and analyzing all the forces that act on a body to determine equilibrium according to Newton's first law or acceleration according to Newton's second law.

CONCEPTUAL QUESTIONS

5.1 Forces

1. What properties do forces have that allow us to classify them as vectors?

5.2 Newton's First Law

2. Taking a frame attached to Earth as inertial, which of the following objects cannot have inertial frames attached to them, and which are inertial reference frames?

(a) A car moving at constant velocity

(b) A car that is accelerating

(c) An elevator in free fall

(d) A space capsule orbiting Earth

(e) An elevator descending uniformly

3. A woman was transporting an open box of cupcakes to a school party. The car in front of her stopped suddenly; she applied her brakes immediately. She was wearing her seat belt and suffered no physical harm (just a great deal of embarrassment), but the cupcakes flew into the dashboard and became “smushcakes.” Explain what happened.

5.3 Newton's Second Law

4. Why can we neglect forces such as those holding a body together when we apply Newton’s second law?

5. A rock is thrown straight up. At the top of the trajectory, the velocity is momentarily zero. Does this imply that the force acting on the object is zero? Explain your answer.

5.4 Mass and Weight

6. What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

7. How much does a 70-kg astronaut weight in space, far from any celestial body? What is her mass at this location?

8. Which of the following statements is accurate?

(a) Mass and weight are the same thing expressed in different units.

(b) If an object has no weight, it must have no mass.

(c) If the weight of an object varies, so must the mass.

(d) Mass and inertia are different concepts.

(e) Weight is always proportional to mass.

9. When you stand on Earth, your feet push against it with a force equal to your weight. Why doesn’t Earth accelerate away from you?

10. How would you give the value of $\vec{\mathbf{g}}$ in vector form?

PROBLEMS

5.1 Forces

19. Two ropes are attached to a tree, and forces of $\vec{\mathbf{F}}_1 = 2.0\,\hat{\mathbf{i}} + 4.0\,\hat{\mathbf{j}}\text{ N}$ and $\vec{\mathbf{F}}_2 = 3.0\,\hat{\mathbf{i}} + 6.0\,\hat{\mathbf{j}}\text{ N}$ are applied. The forces are coplanar (in the same plane). (a) What is the resultant (net force) of these two force vectors? (b) Find the magnitude and direction of this net force.

5.5 Newton’s Third Law

11. Identify the action and reaction forces in the following situations: (a) Earth attracts the Moon, (b) a boy kicks a football, (c) a rocket accelerates upward, (d) a car accelerates forward, (e) a high jumper leaps, and (f) a bullet is shot from a gun.

12. Suppose that you are holding a cup of coffee in your hand. Identify all forces on the cup and the reaction to each force.

13. (a) Why does an ordinary rifle recoil (kick backward) when fired? (b) The barrel of a recoilless rifle is open at both ends. Describe how Newton’s third law applies when one is fired. (c) Can you safely stand close behind one when it is fired?

5.6 Common Forces

14. A table is placed on a rug. Then a book is placed on the table. What does the floor exert a normal force on?

15. A particle is moving to the right. (a) Can the force on it to be acting to the left? If yes, what would happen? (b) Can that force be acting downward? If yes, why?

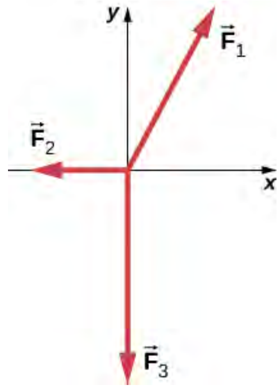
5.7 Drawing Free-Body Diagrams

16. In completing the solution for a problem involving forces, what do we do after constructing the free-body diagram? That is, what do we apply?

17. If a book is located on a table, how many forces should be shown in a free-body diagram of the book? Describe them.

18. If the book in the previous question is in free fall, how many forces should be shown in a free-body diagram of the book? Describe them.

20. A telephone pole has three cables pulling as shown from above, with $\vec{\mathbf{F}}_1 = \left(300.0\,\hat{\mathbf{i}} + 500.0\,\hat{\mathbf{j}}\right)$, $\vec{\mathbf{F}}_2 = -200.0\,\hat{\mathbf{i}}$, and $\vec{\mathbf{F}}_3 = -800.0\,\hat{\mathbf{j}}$. (a) Find the net force on the telephone pole in component form. (b) Find the magnitude and direction of this net force.



21. Two teenagers are pulling on ropes attached to a tree. The angle between the ropes is $30.0°$. David pulls with a force of 400.0 N and Stephanie pulls with a force of 300.0 N. (a) Find the component form of the net force. (b) Find the magnitude of the resultant (net) force on the tree and the angle it makes with David's rope.

5.2 Newton's First Law

22. Two forces of $\vec{\mathbf{F}}_1 = \frac{75.0}{\sqrt{2}}\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)\text{N}$ and $\vec{\mathbf{F}}_2 = \frac{150.0}{\sqrt{2}}\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)\text{N}$ act on an object. Find the third force $\vec{\mathbf{F}}_3$ that is needed to balance the first two forces.

23. While sliding a couch across a floor, Andrea and Jennifer exert forces $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_J$ on the couch. Andrea's force is due north with a magnitude of 130.0 N and Jennifer's force is $32°$ east of north with a magnitude of 180.0 N. (a) Find the net force in component form. (b) Find the magnitude and direction of the net force. (c) If Andrea and Jennifer's housemates, David and Stephanie, disagree with the move and want to prevent its relocation, with what combined force $\vec{\mathbf{F}}_{DS}$ should they push so that the couch does not move?

5.3 Newton's Second Law

24. Andrea, a 63.0-kg sprinter, starts a race with an acceleration of $4.200\ \text{m/s}^2$. What is the net external force on her?

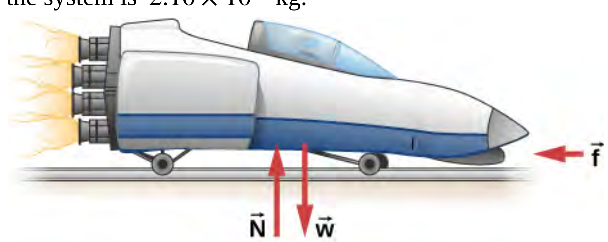
25. If the sprinter from the previous problem accelerates at that rate for 20.00 m and then maintains that velocity for the remainder of a 100.00-m dash, what will her time be for the race?

26. A cleaner pushes a 4.50-kg laundry cart in such a way that the net external force on it is 60.0 N. Calculate the magnitude of his cart's acceleration.

27. Astronauts in orbit are apparently weightless. This means that a clever method of measuring the mass of astronauts is needed to monitor their mass gains or losses, and adjust their diet. One way to do this is to exert a known force on an astronaut and measure the acceleration produced. Suppose a net external force of 50.0 N is exerted, and an astronaut's acceleration is measured to be $0.893\ \text{m/s}^2$. (a) Calculate her mass. (b) By exerting a force on the astronaut, the vehicle in which she orbits experiences an equal and opposite force. Use this knowledge to find an equation for the acceleration of the system (astronaut and spaceship) that would be measured by a nearby observer. (c) Discuss how this would affect the measurement of the astronaut's acceleration. Propose a method by which recoil of the vehicle is avoided.

28. In **Figure 5.12**, the net external force on the 24-kg mower is given as 51 N. If the force of friction opposing the motion is 24 N, what force *F* (in newtons) is the person exerting on the mower? Suppose the mower is moving at 1.5 m/s when the force *F* is removed. How far will the mower go before stopping?

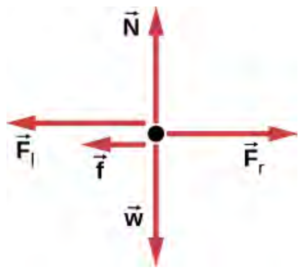
29. The rocket sled shown below decelerates at a rate of $196\ \text{m/s}^2$. What force is necessary to produce this deceleration? Assume that the rockets are off. The mass of the system is 2.10×10^3 kg.



30. If the rocket sled shown in the previous problem starts with only one rocket burning, what is the magnitude of this acceleration? Assume that the mass of the system is 2.10×10^3 kg, the thrust T is 2.40×10^4 N, and the force of friction opposing the motion is 650.0 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?

31. What is the deceleration of the rocket sled if it comes to rest in 1.10 s from a speed of 1000.0 km/h? (Such deceleration caused one test subject to black out and have temporary blindness.)

32. Suppose two children push horizontally, but in exactly opposite directions, on a third child in a wagon. The first child exerts a force of 75.0 N, the second exerts a force of 90.0 N, friction is 12.0 N, and the mass of the third child plus wagon is 23.0 kg. (a) What is the system of interest if the acceleration of the child in the wagon is to be calculated? (See the free-body diagram.) (b) Calculate the acceleration. (c) What would the acceleration be if friction were 15.0 N?



33. A powerful motorcycle can produce an acceleration of $3.50\,\text{m/s}^2$ while traveling at 90.0 km/h. At that speed, the forces resisting motion, including friction and air resistance, total 400.0 N. (Air resistance is analogous to air friction. It always opposes the motion of an object.) What is the magnitude of the force that motorcycle exerts backward on the ground to produce its acceleration if the mass of the motorcycle with rider is 245 kg?

34. A car with a mass of 1000.0 kg accelerates from 0 to 90.0 km/h in 10.0 s. (a) What is its acceleration? (b) What is the net force on the car?

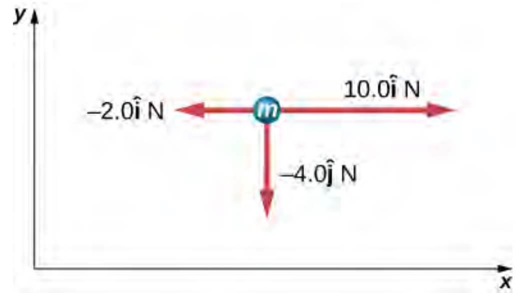
35. The driver in the previous problem applies the brakes when the car is moving at 90.0 km/h, and the car comes to rest after traveling 40.0 m. What is the net force on the car during its deceleration?

36. An 80.0-kg passenger in an SUV traveling at 1.00×10^3 km/h is wearing a seat belt. The driver slams on the brakes and the SUV stops in 45.0 m. Find the force of the seat belt on the passenger.

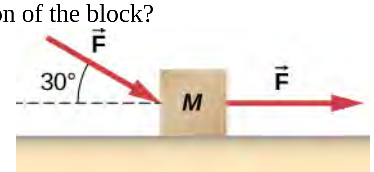
37. A particle of mass 2.0 kg is acted on by a single force $\vec{\mathbf{F}}_1 = 18\,\hat{\mathbf{i}}$ N. (a) What is the particle's acceleration? (b) If the particle starts at rest, how far does it travel in the first 5.0 s?

38. Suppose that the particle of the previous problem also experiences forces $\vec{\mathbf{F}}_2 = -15\,\hat{\mathbf{i}}$ N and $\vec{\mathbf{F}}_3 = 6.0\,\hat{\mathbf{j}}$ N. What is its acceleration in this case?

39. Find the acceleration of the body of mass 5.0 kg shown below.



40. In the following figure, the horizontal surface on which this block slides is frictionless. If the two forces acting on it each have magnitude $F = 30.0\,\text{N}$ and $M = 10.0\,\text{kg}$, what is the magnitude of the resulting acceleration of the block?



5.4 Mass and Weight

41. The weight of an astronaut plus his space suit on the Moon is only 250 N. (a) How much does the suited astronaut weigh on Earth? (b) What is the mass on the Moon? On Earth?

42. Suppose the mass of a fully loaded module in which astronauts take off from the Moon is 1.00×10^4 kg. The thrust of its engines is 3.00×10^4 N. (a) Calculate the module's magnitude of acceleration in a vertical takeoff from the Moon. (b) Could it lift off from Earth? If not, why not? If it could, calculate the magnitude of its acceleration.

43. A rocket sled accelerates at a rate of $49.0\,\text{m/s}^2$. Its passenger has a mass of 75.0 kg. (a) Calculate the horizontal component of the force the seat exerts against his body. Compare this with his weight using a ratio. (b) Calculate the direction and magnitude of the total force the seat exerts against his body.

44. Repeat the previous problem for a situation in which the rocket sled decelerates at a rate of $201\,\text{m/s}^2$. In this problem, the forces are exerted by the seat and the seat belt.

45. A body of mass 2.00 kg is pushed straight upward by a 25.0 N vertical force. What is its acceleration?

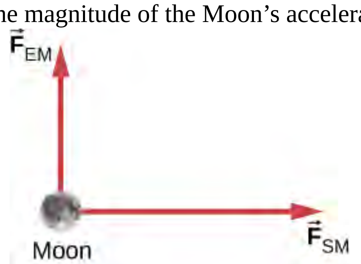
46. A car weighing 12,500 N starts from rest and accelerates to 83.0 km/h in 5.00 s. The friction force is 1350 N. Find the applied force produced by the engine.

47. A body with a mass of 10.0 kg is assumed to be in Earth's gravitational field with $g = 9.80 \text{ m/s}^2$. What is its acceleration?

48. A fireman has mass *m*; he hears the fire alarm and slides down the pole with acceleration *a* (which is less than *g* in magnitude). (a) Write an equation giving the vertical force he must apply to the pole. (b) If his mass is 90.0 kg and he accelerates at 5.00 m/s^2, what is the magnitude of his applied force?

49. A baseball catcher is performing a stunt for a television commercial. He will catch a baseball (mass 145 g) dropped from a height of 60.0 m above his glove. His glove stops the ball in 0.0100 s. What is the force exerted by his glove on the ball?

50. When the Moon is directly overhead at sunset, the force by Earth on the Moon, F_{EM}, is essentially at $90°$ to the force by the Sun on the Moon, F_{SM}, as shown below. Given that $F_{\text{EM}} = 1.98 \times 10^{20} \text{ N}$ and $F_{\text{SM}} = 4.36 \times 10^{20} \text{ N}$, all other forces on the Moon are negligible, and the mass of the Moon is $7.35 \times 10^{22} \text{ kg}$, determine the magnitude of the Moon's acceleration.

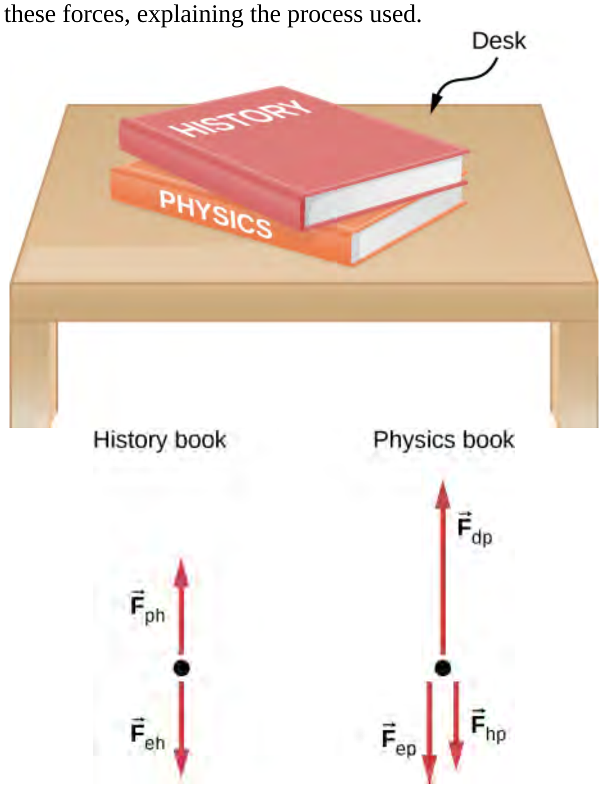


5.5 Newton's Third Law

51. (a) What net external force is exerted on a 1100.0-kg artillery shell fired from a battleship if the shell is accelerated at $2.40 \times 10^4 \text{ m/s}^2$? (b) What is the magnitude of the force exerted on the ship by the artillery shell, and why?

52. A brave but inadequate rugby player is being pushed backward by an opposing player who is exerting a force of 800.0 N on him. The mass of the losing player plus equipment is 90.0 kg, and he is accelerating backward at 1.20 m/s^2. (a) What is the force of friction between the losing player's feet and the grass? (b) What force does the winning player exert on the ground to move forward if his mass plus equipment is 110.0 kg?

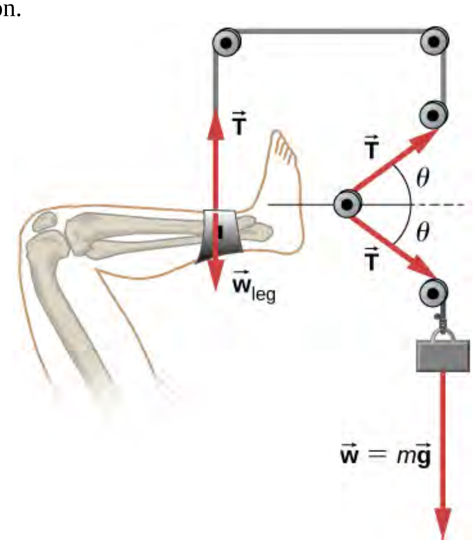
53. A history book is lying on top of a physics book on a desk, as shown below; a free-body diagram is also shown. The history and physics books weigh 14 N and 18 N, respectively. Identify each force on each book with a double subscript notation (for instance, the contact force of the history book pressing against physics book can be described as $\vec{\mathbf{F}}_{\text{HP}}$), and determine the value of each of these forces, explaining the process used.



54. A truck collides with a car, and during the collision, the net force on each vehicle is essentially the force exerted by the other. Suppose the mass of the car is 550 kg, the mass of the truck is 2200 kg, and the magnitude of the truck's acceleration is 10 m/s^2. Find the magnitude of the car's acceleration.

5.6 Common Forces

55. A leg is suspended in a traction system, as shown below. (a) Which part of the figure is used to calculate the force exerted on the foot? (b) What is the tension in the rope? Here $\vec{\mathbf{T}}$ is the tension, $\vec{\mathbf{w}}_{\text{leg}}$ is the weight of the leg, and $\vec{\mathbf{w}}$ is the weight of the load that provides the tension.



56. Suppose the shinbone in the preceding image was a femur in a traction setup for a broken bone, with pulleys and rope available. How might we be able to increase the force along the femur using the same weight?

57. Two teams of nine members each engage in tug-of-war. Each of the first team's members has an average mass of 68 kg and exerts an average force of 1350 N horizontally. Each of the second team's members has an average mass of 73 kg and exerts an average force of 1365 N horizontally. (a) What is magnitude of the acceleration of the two teams, and which team wins? (b) What is the tension in the section of rope between the teams?

58. What force does a trampoline have to apply to Jennifer, a 45.0-kg gymnast, to accelerate her straight up at $7.50\ \text{m/s}^2$? The answer is independent of the velocity of the gymnast—she can be moving up or down or can be instantly stationary.

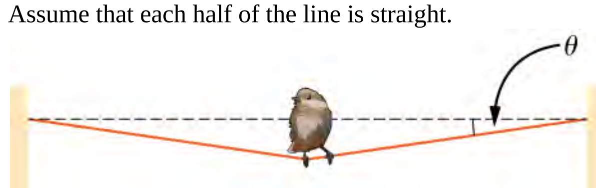
59. (a) Calculate the tension in a vertical strand of spider web if a spider of mass 2.00×10^{-5} kg hangs motionless on it. (b) Calculate the tension in a horizontal strand of spider web if the same spider sits motionless in the middle of it much like the tightrope walker in Figure 5.26. The strand sags at an angle of $12°$ below the horizontal. Compare this with the tension in the vertical strand (find their ratio).

60. Suppose Kevin, a 60.0-kg gymnast, climbs a rope. (a) What is the tension in the rope if he climbs at a constant speed? (b) What is the tension in the rope if he accelerates upward at a rate of $1.50\ \text{m/s}^2$?

61. Show that, as explained in the text, a force $F_\perp$ exerted on a flexible medium at its center and perpendicular to its length (such as on the tightrope wire in Figure 5.26) gives rise to a tension of magnitude $T = F_\perp\ /2\sin(\theta)$.

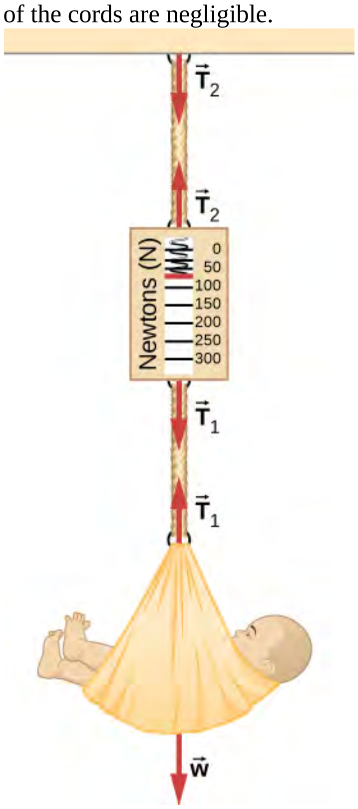
62. Consider Figure 5.28. The driver attempts to get the car out of the mud by exerting a perpendicular force of 610.0 N, and the distance she pushes in the middle of the rope is 1.00 m while she stands 6.00 m away from the car on the left and 6.00 m away from the tree on the right. What is the tension T in the rope, and how do you find the answer?

63. A bird has a mass of 26 g and perches in the middle of a stretched telephone line. (a) Show that the tension in the line can be calculated using the equation $T = \dfrac{mg}{2\sin\theta}$. Determine the tension when (b) $\theta = 5°$ and (c) $\theta = 0.5°$. Assume that each half of the line is straight.

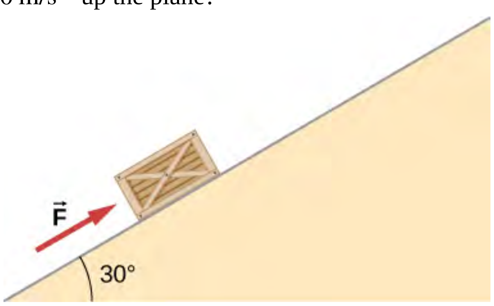


64. One end of a 30-m rope is tied to a tree; the other end is tied to a car stuck in the mud. The motorist pulls sideways on the midpoint of the rope, displacing it a distance of 2 m. If he exerts a force of 80 N under these conditions, determine the force exerted on the car.

65. Consider the baby being weighed in the following figure. (a) What is the mass of the infant and basket if a scale reading of 55 N is observed? (b) What is tension T_1 in the cord attaching the baby to the scale? (c) What is tension T_2 in the cord attaching the scale to the ceiling, if the scale has a mass of 0.500 kg? (d) Sketch the situation, indicating the system of interest used to solve each part. The masses of the cords are negligible.



66. What force must be applied to a 100.0-kg crate on a frictionless plane inclined at $30°$ to cause an acceleration of 2.0 m/s^2 up the plane?



67. A 2.0-kg block is on a perfectly smooth ramp that makes an angle of $30°$ with the horizontal. (a) What is the block's acceleration down the ramp and the force of the ramp on the block? (b) What force applied upward along and parallel to the ramp would allow the block to move with constant velocity?

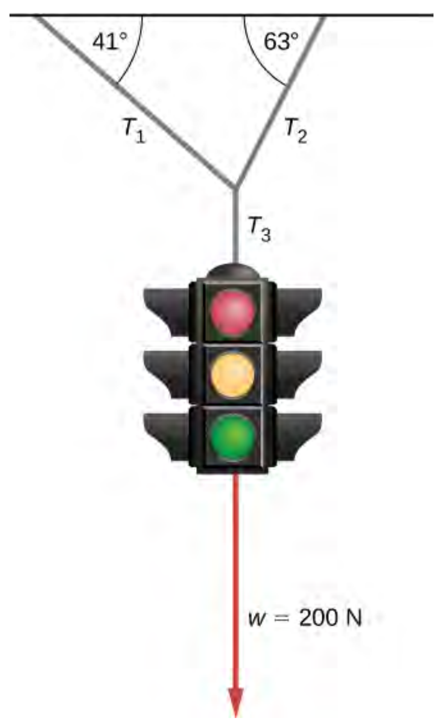
5.7 Drawing Free-Body Diagrams

68. A ball of mass *m* hangs at rest, suspended by a string. (a) Sketch all forces. (b) Draw the free-body diagram for the ball.

69. A car moves along a horizontal road. Draw a free-body diagram; be sure to include the friction of the road that opposes the forward motion of the car.

70. A runner pushes against the track, as shown. (a) Provide a free-body diagram showing all the forces on the runner. (*Hint:* Place all forces at the center of his body, and include his weight.) (b) Give a revised diagram showing the *xy*-component form.

71. The traffic light hangs from the cables as shown. Draw a free-body diagram on a coordinate plane for this situation.



ADDITIONAL PROBLEMS

72. Two small forces, $\vec{\mathbf{F}}_1 = -2.40\,\hat{\mathbf{i}} - 6.10t\,\hat{\mathbf{j}}$ N and $\vec{\mathbf{F}}_2 = 8.50\,\hat{\mathbf{i}} - 9.70\,\hat{\mathbf{j}}$ N, are exerted on a rogue asteroid by a pair of space tractors. (a) Find the net force. (b) What are the magnitude and direction of the net force? (c) If the mass of the asteroid is 125 kg, what acceleration does it experience (in vector form)? (d) What are the magnitude and direction of the acceleration?

73. Two forces of 25 and 45 N act on an object. Their directions differ by $70°$. The resulting acceleration has magnitude of $10.0\,\text{m/s}^2$. What is the mass of the body?

74. A force of 1600 N acts parallel to a ramp to push a 300-kg piano into a moving van. The ramp is inclined at $20°$. (a) What is the acceleration of the piano up the ramp? (b) What is the velocity of the piano when it reaches the top if the ramp is 4.0 m long and the piano starts from rest?

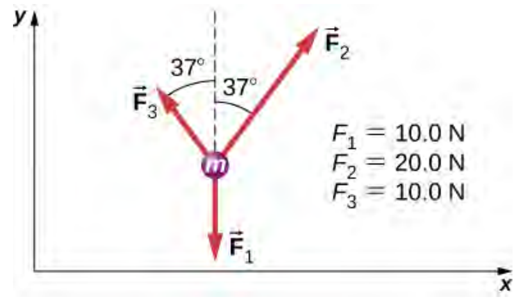
75. Draw a free-body diagram of a diver who has entered the water, moved downward, and is acted on by an upward force due to the water which balances the weight (that is, the diver is suspended).

76. For a swimmer who has just jumped off a diving board, assume air resistance is negligible. The swimmer has a mass of 80.0 kg and jumps off a board 10.0 m above the water. Three seconds after entering the water, her downward motion is stopped. What average upward force did the water exert on her?

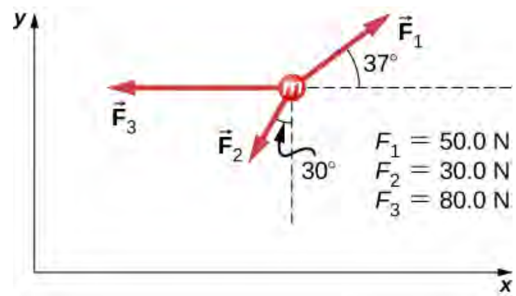
77. (a) Find an equation to determine the magnitude of the net force required to stop a car of mass *m*, given that the initial speed of the car is v_0 and the stopping distance is *x*. (b) Find the magnitude of the net force if the mass of the car is 1050 kg, the initial speed is 40.0 km/h, and the stopping distance is 25.0 m.

78. A sailboat has a mass of 1.50×10^3 kg and is acted on by a force of 2.00×10^3 N toward the east, while the wind acts behind the sails with a force of 3.00×10^3 N in a direction $45°$ north of east. Find the magnitude and direction of the resulting acceleration.

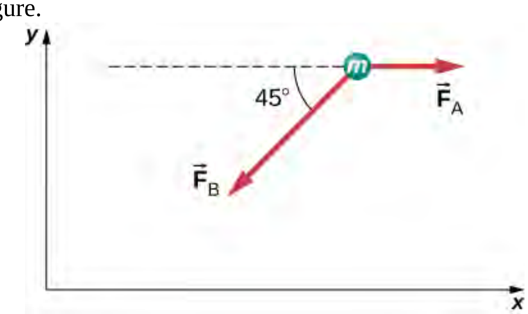
79. Find the acceleration of the body of mass 10.0 kg shown below.



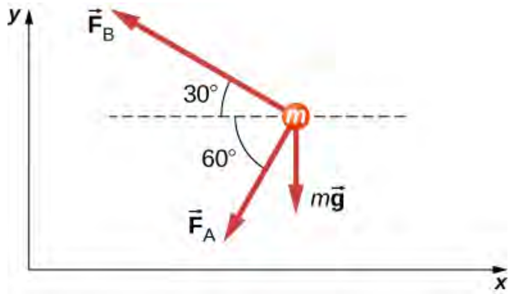
80. A body of mass 2.0 kg is moving along the *x*-axis with a speed of 3.0 m/s at the instant represented below. (a) What is the acceleration of the body? (b) What is the body's velocity 10.0 s later? (c) What is its displacement after 10.0 s?



81. Force $\vec{\mathbf{F}}_B$ has twice the magnitude of force $\vec{\mathbf{F}}_A$. Find the direction in which the particle accelerates in this figure.



82. Shown below is a body of mass 1.0 kg under the influence of the forces $\vec{\mathbf{F}}_{\mathbf{A}}$, $\vec{\mathbf{F}}_{\mathbf{B}}$, and $m\vec{\mathbf{g}}$. If the body accelerates to the left at $0.20\ \text{m/s}^2$, what are $\vec{\mathbf{F}}_{\mathbf{A}}$ and $\vec{\mathbf{F}}_{\mathbf{B}}$?



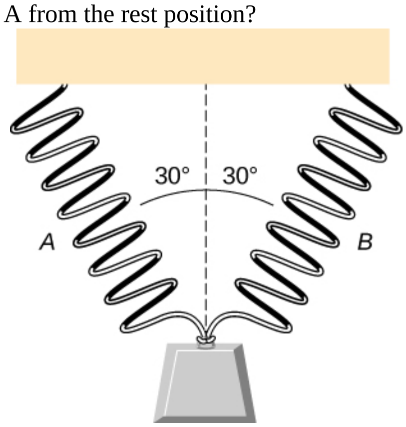
83. A force acts on a car of mass *m* so that the speed *v* of the car increases with position *x* as $v = kx^2$, where *k* is constant and all quantities are in SI units. Find the force acting on the car as a function of position.

84. A 7.0-N force parallel to an incline is applied to a 1.0-kg crate. The ramp is tilted at $20°$ and is frictionless. (a) What is the acceleration of the crate? (b) If all other conditions are the same but the ramp has a friction force of 1.9 N, what is the acceleration?

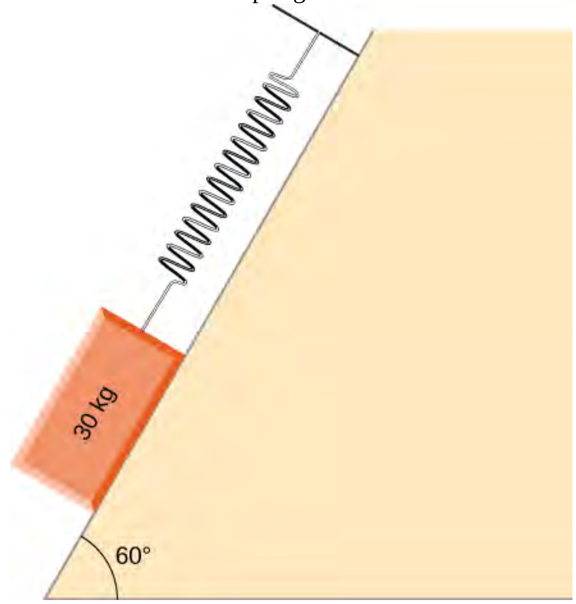
85. Two boxes, A and B, are at rest. Box A is on level ground, while box B rests on an inclined plane tilted at angle θ with the horizontal. (a) Write expressions for the normal force acting on each block. (b) Compare the two forces; that is, tell which one is larger or whether they are equal in magnitude. (c) If the angle of incline is $10°$, which force is greater?

86. A mass of 250.0 g is suspended from a spring hanging vertically. The spring stretches 6.00 cm. How much will the spring stretch if the suspended mass is 530.0 g?

87. As shown below, two identical springs, each with the spring constant 20 N/m, support a 15.0-N weight. (a) What is the tension in spring A? (b) What is the amount of stretch of spring A from the rest position?

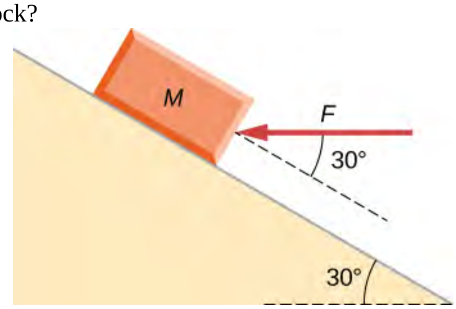


88. Shown below is a 30.0-kg block resting on a frictionless ramp inclined at $60°$ to the horizontal. The block is held by a spring that is stretched 5.0 cm. What is the force constant of the spring?



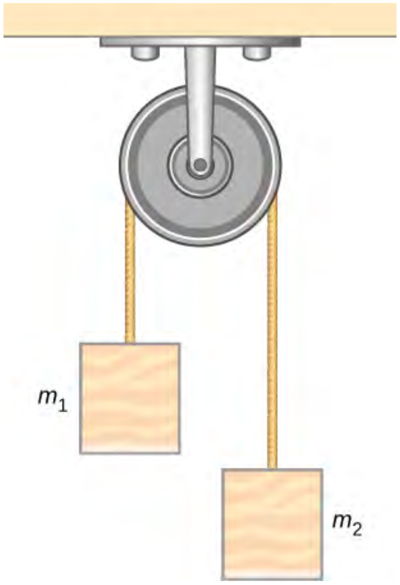
89. In building a house, carpenters use nails from a large box. The box is suspended from a spring twice during the day to measure the usage of nails. At the beginning of the day, the spring stretches 50 cm. At the end of the day, the spring stretches 30 cm. What fraction or percentage of the nails have been used?

90. A force is applied to a block to move it up a $30°$ incline. The incline is frictionless. If $F = 65.0\ \text{N}$ and $M = 5.00\ \text{kg}$, what is the magnitude of the acceleration of the block?



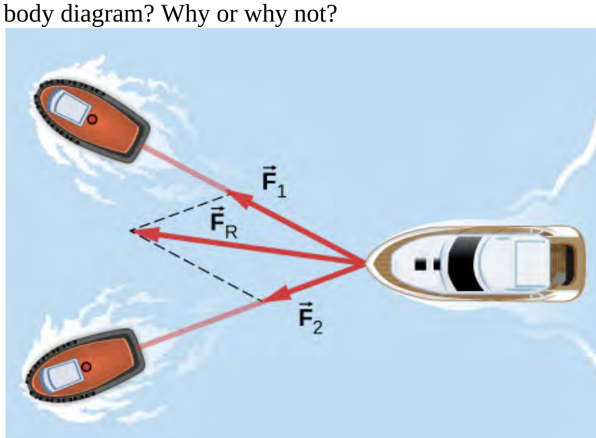
91. Two forces are applied to a 5.0-kg object, and it accelerates at a rate of $2.0\ \text{m/s}^2$ in the positive *y*-direction. If one of the forces acts in the positive *x*-direction with magnitude 12.0 N, find the magnitude of the other force.

92. The block on the right shown below has more mass than the block on the left ($m_2 > m_1$). Draw free-body diagrams for each block.



CHALLENGE PROBLEMS

93. If two tugboats pull on a disabled vessel, as shown here in an overhead view, the disabled vessel will be pulled along the direction indicated by the result of the exerted forces. (a) Draw a free-body diagram for the vessel. Assume no friction or drag forces affect the vessel. (b) Did you include all forces in the overhead view in your free-body diagram? Why or why not?



94. A 10.0-kg object is initially moving east at 15.0 m/s. Then a force acts on it for 2.00 s, after which it moves northwest, also at 15.0 m/s. What are the magnitude and direction of the average force that acted on the object over the 2.00-s interval?

95. On June 25, 1983, shot-putter Udo Beyer of East Germany threw the 7.26-kg shot 22.22 m, which at that time was a world record. (a) If the shot was released at a height of 2.20 m with a projection angle of $45.0°$, what was its initial velocity? (b) If while in Beyer's hand the shot was accelerated uniformly over a distance of 1.20 m, what was the net force on it?

96. A body of mass *m* moves in a horizontal direction such that at time *t* its position is given by $x(t) = at^4 + bt^3 + ct,$ where *a*, *b*, and *c* are constants. (a) What is the acceleration of the body? (b) What is the time-dependent force acting on the body?

97. A body of mass *m* has initial velocity v_0 in the positive *x*-direction. It is acted on by a constant force *F* for time *t* until the velocity becomes zero; the force continues to act on the body until its velocity becomes $-v_0$ in the same amount of time. Write an expression for the total distance the body travels in terms of the variables indicated.

98. The velocities of a 3.0-kg object at $t = 6.0\text{ s}$ and $t = 8.0\text{ s}$ are $(3.0\,\hat{\mathbf{i}} - 6.0\,\hat{\mathbf{j}} + 4.0\,\hat{\mathbf{k}})\text{ m/s}$ and $(-2.0\,\hat{\mathbf{i}} + 4.0\,\hat{\mathbf{k}})\text{ m/s}$, respectively. If the object is moving at constant acceleration, what is the force acting on it?

99. A 120-kg astronaut is riding in a rocket sled that is sliding along an inclined plane. The sled has a horizontal component of acceleration of 5.0 m/s^2 and a downward component of 3.8 m/s^2. Calculate the magnitude of the force on the rider by the sled. (*Hint*: Remember that gravitational acceleration must be considered.)

100. Two forces are acting on a 5.0-kg object that moves with acceleration 2.0 m/s^2 in the positive *y*-direction. If one of the forces acts in the positive *x*-direction and has magnitude of 12 N, what is the magnitude of the other force?

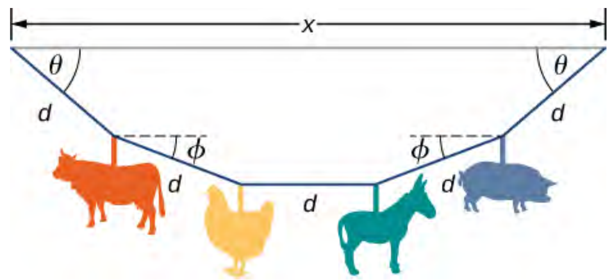
101. Suppose that you are viewing a soccer game from a helicopter above the playing field. Two soccer players simultaneously kick a stationary soccer ball on the flat field; the soccer ball has mass 0.420 kg. The first player kicks with force 162 N at $9.0°$ north of west. At the same instant, the second player kicks with force 215 N at $15°$ east of south. Find the acceleration of the ball in $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ form.

102. A 10.0-kg mass hangs from a spring that has the spring constant 535 N/m. Find the position of the end of the spring away from its rest position. (Use $g = 9.80 \text{ m/s}^2$.)

103. A 0.0502-kg pair of fuzzy dice is attached to the rearview mirror of a car by a short string. The car accelerates at constant rate, and the dice hang at an angle of $3.20°$ from the vertical because of the car's acceleration. What is the magnitude of the acceleration of the car?

104. At a circus, a donkey pulls on a sled carrying a small clown with a force given by $2.48\hat{\mathbf{i}} + 4.33\hat{\mathbf{j}} \text{ N}$. A horse pulls on the same sled, aiding the hapless donkey, with a force of $6.56\hat{\mathbf{i}} + 5.33\hat{\mathbf{j}} \text{ N}$. The mass of the sled is 575 kg. Using $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ form for the answer to each problem, find (a) the net force on the sled when the two animals act together, (b) the acceleration of the sled, and (c) the velocity after 6.50 s.

105. Hanging from the ceiling over a baby bed, well out of baby's reach, is a string with plastic shapes, as shown here. The string is taut (there is no slack), as shown by the straight segments. Each plastic shape has the same mass *m*, and they are equally spaced by a distance *d*, as shown. The angles labeled θ describe the angle formed by the end of the string and the ceiling at each end. The center length of sting is horizontal. The remaining two segments each form an angle with the horizontal, labeled ϕ. Let T_1 be the tension in the leftmost section of the string, T_2 be the tension in the section adjacent to it, and T_3 be the tension in the horizontal segment. (a) Find an equation for the tension in each section of the string in terms of the variables *m*, *g*, and θ. (b) Find the angle ϕ in terms of the angle θ. (c) If $\theta = 5.10°$, what is the value of ϕ? (d) Find the distance *x* between the endpoints in terms of *d* and θ.



106. A bullet shot from a rifle has mass of 10.0 g and travels to the right at 350 m/s. It strikes a target, a large bag of sand, penetrating it a distance of 34.0 cm. Find the magnitude and direction of the retarding force that slows and stops the bullet.

107. An object is acted on by three simultaneous forces: $\vec{\mathbf{F}}_1 = \left(-3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}\right)\text{N}$, $\vec{\mathbf{F}}_2 = \left(6.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right)\text{N}$, and $\vec{\mathbf{F}}_3 = \left(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\text{N}$. The object experiences acceleration of 4.23 m/s^2. (a) Find the acceleration vector in terms of *m*. (b) Find the mass of the object. (c) If the object begins from rest, find its speed after 5.00 s. (d) Find the components of the velocity of the object after 5.00 s.

108. In a particle accelerator, a proton has mass 1.67×10^{-27} kg and an initial speed of 2.00×10^5 m/s. It moves in a straight line, and its speed increases to 9.00×10^5 m/s in a distance of 10.0 cm. Assume that the acceleration is constant. Find the magnitude of the force exerted on the proton.

109. A drone is being directed across a frictionless ice-covered lake. The mass of the drone is 1.50 kg, and its velocity is $3.00\,\hat{\mathbf{i}}\,\text{m/s}$. After 10.0 s, the velocity is $9.00\,\hat{\mathbf{i}} + 4.00\,\hat{\mathbf{j}}\,\text{m/s}$. If a constant force in the horizontal direction is causing this change in motion, find (a) the components of the force and (b) the magnitude of the force.