Using Truth Tables for Sentences

PHIL 2020 Day 15 Week 8

Truth Tables

- Remember the Order of Operations for Truth Tables
- Set up the table by the number of different letters, and lines
- The number of lines will be 2^n, n is the number of different letters

Operators (Both Sets)

Same as:

Hook ¬

Ampersand &

Wedge v

Arrow →

Double Arrow ⇔

Operator	Name	Logical Function	Used to Translate
~	tilde	negation	not, it is false that, it is not the case that
●	dot	conjunction	and, also, but, moreover, however, nevertheless, still, both, additionally, furthermore
v	wedge	disjunction	or, unless
⊃	horseshoe	implication	if...then..., only if, given that, provided that, in case, on condition, that, sufficient condition for, necessary condition for
≡	triple bar	equivalence	if and only if, is a necessary and sufficient condition for

~ • ∨ ⊃ ≡
Truth Functions

p	$\sim p$
T	F
F	T

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Functions written out in English:

~ changes the truth value of whatever it has “scope” over from T to F and from F to T; note that some ~ are on letters and some are on brackets or parentheses.

• is only True if both sides are True, T•T.

V is only False if both sides are False, FvF. If there is even one T on one side, or T on both sides, the v is true.

⊃ is only false in a line where T⊃F, because that’s like a cause happening without the effect. Any other combination of T⊃T, F⊃T, or F⊃F is T for the ⊃.

≡ is T as long as the sides match, T≡T or F≡F is True for the ≡.

Order of Operations when Truth Tabling

~ • v ⊃ ≡

- 1) Tilde ~ on single letters should be done after the letters themselves are already done.
- 2) Connectors inside parentheses should be done next, after each side in the parentheses is done.
- 3) Tilde ~ outside of parentheses should be done next. Make sure you are changing the truth value of the connector column, not a side column.
- 4) Connectors that are in between two sets of parentheses should be done next.
- 5) Tilde ~ on big brackets, larger brackets [] and {}

~ • ∨ ⊃ ≡

Truth Functions (how the operators are true or false)

p	$\sim p$
T	F
F	T

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

Constructing a Truth Table

- The number of lines in the truth table relates to the number of different letters in your sentence.
- 2 to the n power 2^n where n is the number of different letters
- 2 different letters = 2x2 = 4 lines
- 3 different letters = 2x2x2 = 8 lines
- The first letter has half the lines true, and half the lines false.
- The next letter divides those in half again, so if you have an 8 line table with a first letter with four true lines, the second letter will be two true lines, two false lines, and so on.

What are the values of the column for the "dot"?

C	***S***		***C***	**•**	***S***
T	T		T		T
F	T		F		T
T	F		T		F
F	F		F		F

The complete table for our example:

C	*S*		*C*	•	*S*
T	T		T	T	T
F	T		F	F	T
T	F		T	F	F
F	F		F	F	F

The basic truth table for conjunction:

p	q		p	$\bullet$	q
T	T		T	T	T
F	T		F	F	T
T	F		T	F	F
F	F		F	F	F

What are the values in the column for the "*v*"?

C	***S***		***C***	***v***	***S***
T	T		T		T
F	T		F		T
T	F		T		F
F	F		F		F

The complete truth table for our example:

C	***S***		***C***	***v***	***S***
T	T		T	T	T
F	T		F	T	T
T	F		T	T	F
F	F		F	F	F

The basic truth table for disjunction:

p	***q***		***p***	***v***	***q***
T	T		T	T	T
F	T		F	T	T
T	F		T	T	F
F	F		F	F	F

Sample from 6.1 page 334 number 12:

$$A \supset \sim(Z \vee \sim Y)$$

It has three different letters, so $2^3 = 2 \cdot 2 \cdot 2 = 8$ lines

Do the letters first:

A	$\supset$	$\sim$	$(Z$	$\vee$	$\sim Y)$
T			T		T
T			T		F
T			F		T
T			F		F
F			T		T
F			T		F
F			F		T
F			F		F

Then do the ~ on single letters:

A	$\supset$	$\sim$	$(Z$	$\vee$	$\sim$	$Y)$
T			T		F	T
T			T		T	F
T			F		F	T
T			F		T	F
F			T		F	T
F			T		T	F
F			F		F	T
F			F		T	F

in this case,
this is the
one ~ on
a single
letter.
the other ~
is on parentheses.

Then do the connective operator inside the parentheses:

A	⊃	~	(Z	v	~	Y)
T			T	T	F	T
T			T	T	T	F
T			F	F	F	T
T			F	T	T	F
F			T	T	F	T
F			T	T	T	F
F			F	F	F	T
F			F	T	T	F

the sides of the parentheses came from the Z and the ~

Next, do the ~ outside the parentheses:

A	⊃	~	(Z	v	~	Y)
T		F	T	T	F	T
T		F	T	T	T	F
T		T	F	F	F	T
T		F	F	T	T	F
F		F	T	T	F	T
F		F	T	T	T	F
F		T	F	F	F	T
F		F	F	T	T	F

the ~ truth value here comes from the v inside the parentheses

Next, do the main connective, which is the $\supset$

A	$\supset$	$\sim$	$(Z$	$\vee$	$\sim$	$Y)$
T	F	F	T	T	F	T
T	F	F	T	T	T	F
T	T	T	F	F	F	T
T	F	F	F	T	T	F
F	T	F	T	T	F	T
F	T	F	T	T	T	F
F	T	T	F	F	F	T
F	T	F	F	T	T	F

the $\supset$ truth values come from the A and the $\sim$

The main connective (or the main operator) is always the item you would have to do last on the truth table:

page 332

② $(G \cdot \sim P) \supset \sim(H \vee \sim W)$

③ $\sim[P \cdot (S \equiv K)]$

⑤ $(M \cdot B) \vee \sim[E \equiv \sim(C \vee I)]$

⑥ $\sim[(P \cdot \sim R) \supset (\sim E \vee F)]$

6.3 has a special set of terms to describe

SINGLE STATEMENTS

and

PAIRS OF STATEMENTS

based on how the truth table looks
when it is filled in.

A single statement that is true on every line of its truth table underneath its main operator is TAUTOLOGOUS

For example:

$$[(A \supset B) \cdot A] \supset B$$

A	$\supset$	B	$\cdot$	A	$\supset$	B
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	T	F	F	F	T	F

main operator all T: TAUTOLOGOUS

the As should match all the way up and down, and all the Bs match each other too.

If a single statement is false on every line, it is SELF-CONTRADICTORY

$(P \cdot {\sim}Q) \equiv (P \supset Q)$

P	$\cdot$	~	Q	$\equiv$	P	$\supset$	Q
T	F	F	T	F	T	T	T
T	T	T	F	F	T	F	F
F	F	F	T	F	F	T	T
F	F	T	F	F	F	T	F

main operator always F
SELF-CONTRADICTORY

Most single statements will be a mix of true and false; CONTINGENT

$(C \vee D) \cdot (D \supset C)$

C	$\vee$	D	$\cdot$	D	$\supset$	C
T	T	T	T	T	T	T
T	T	F	T	F	T	T
F	T	T	F	T	F	F
F	F	F	F	F	T	F

mix of TF
CONTINGENT

When comparing a pair of statements, keep the tables consistent and make sure the same letters match in both sentences (all the As are the same, all the Ps are the same, etc.)

Same truthvalue on each line under main operators = LOGICALLY EQUIVALENT

T	T
T	T
F	F
F	F

Never the same truth values under main operators = CONTRADICTORY

T	F
F	T
T	F
T	F

At least one line T T across = CONSISTENT

Never even one line T T across = INCONSISTENT

LOGICALLY EQUIVALENT:

P	$\supset$	Q
T	T	T
T	F	F
F	T	T
F	T	F

$\sim$	P	$\vee$	Q
F	T	T	T
F	T	F	F
T	F	T	T
T	F	T	F

CONTRADICTORY:

A	$\bullet$	$\sim$	B
T	F	F	T
T	T	T	F
F	F	F	T
F	F	T	F

A	$\supset$	B
T	T	T
T	F	F
F	T	T
F	T	F

CONSISTENT:

$\sim$	K	$\supset$	L
F	T	T	T
F	T	T	F
T	F	T	T
T	F	F	F

K	$\supset$	$\sim$	L
T	F	F	T
T	T	T	F
F	T	F	T
F	T	T	F

in these two lines, they are T T ~~true~~ together across the table under the main connectors

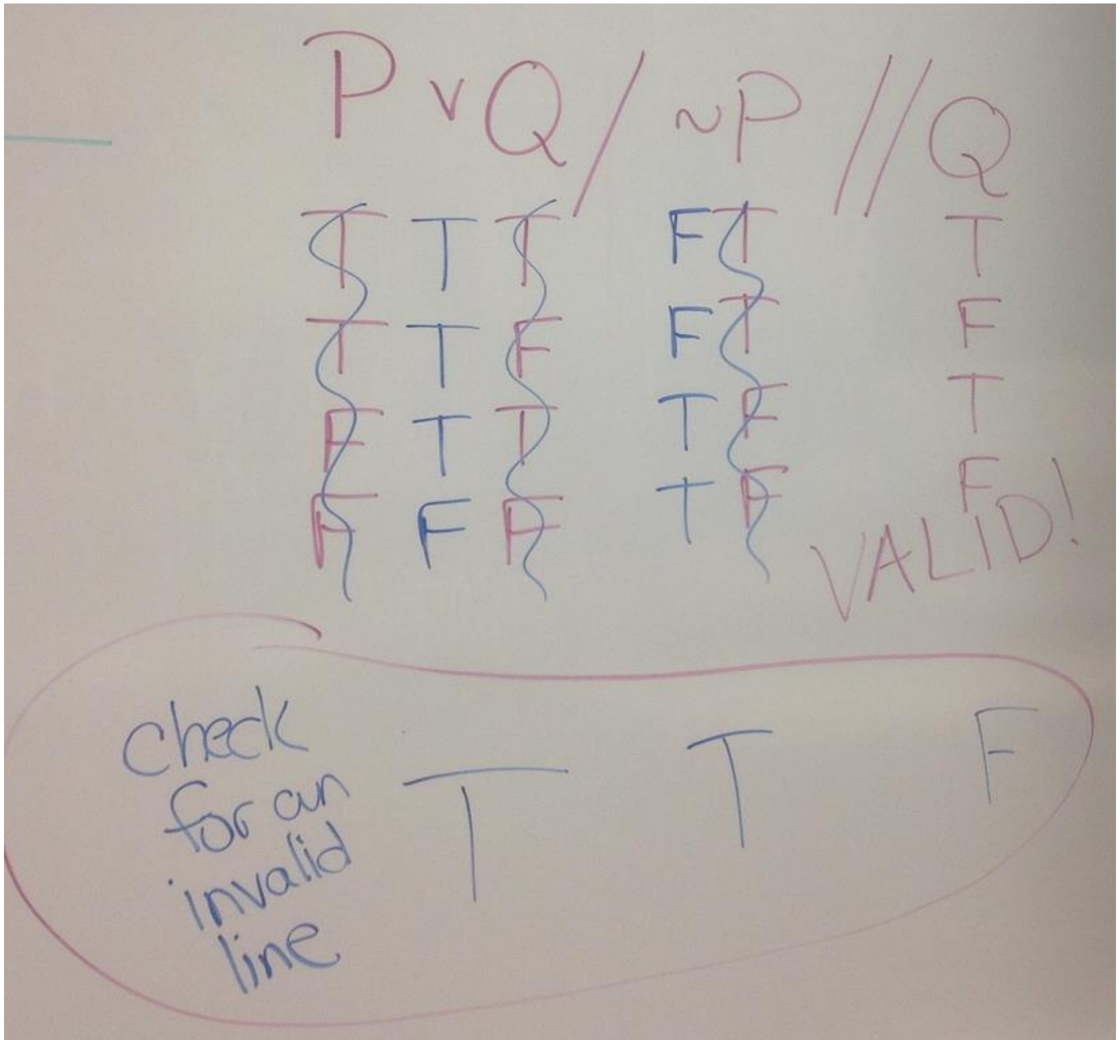
INCONSISTENT:

F	$\bullet$	M
T	T	T
T	F	F
F	F	T
F	F	F

$\sim$	(F	$\vee$	M)
F	T	T	T
F	T	T	F
F	F	T	T
T	F	F	F

notice there is never
a line where they
are T T
in the same line
under the main
connectors.

Validity tests for arguments using truth tables



This argument is valid, there is no TT->F line with all true premises and a false conclusion

This argument is invalid; there is a line with all true premises and a false conclusion