

Kinematics

Objective: Learning about the fundamental concepts used to describe motion – “Kinematics”

- Kinematics – 'How'
- Dynamics – 'Why' (Forces)

Key concepts:

- ❖ 1-d Motion/Kinematics
- ❖ Displacement
- ❖ Velocity
- ❖ Acceleration
- ❖ Graphing Motion

Position, Displacement and Distance

Need coordinate system:

- Axes (we've chosen +x to right & +y up)
- Origin defined as $(x,y) = (0,0)$

➤ **“Position”**: Location relative to origin

Initial position: $x_0 = +1.5\text{m}$

Final position: $x_f = +3.5\text{m}$

For both, $y_f = y_i = 0$

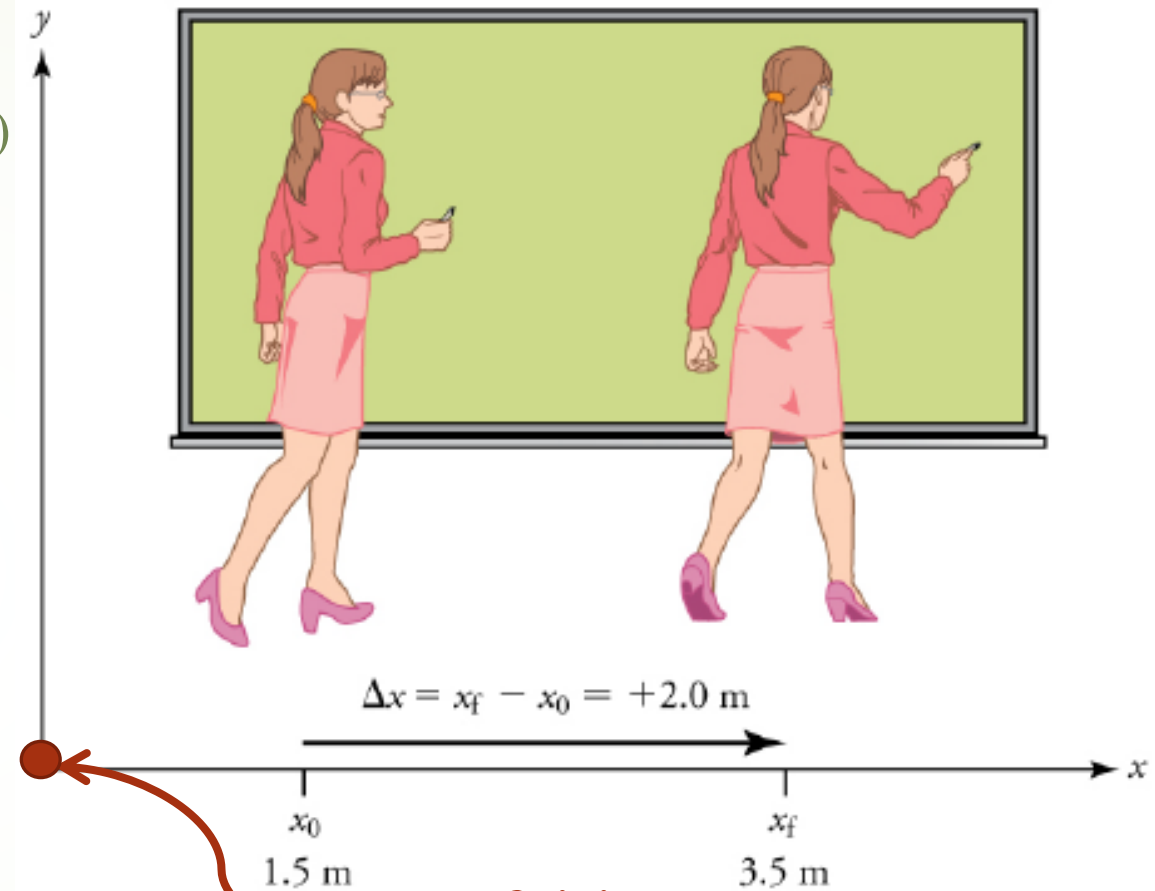
➤ **“Displacement”**: Change in Position

$$\Delta x = x_f - x_0$$

Displacement would be 2.0m to the right
(Greek letter delta, Δ = change = final – initial)

➤ **“Distance”**: Ignore direction and add up distance traveled

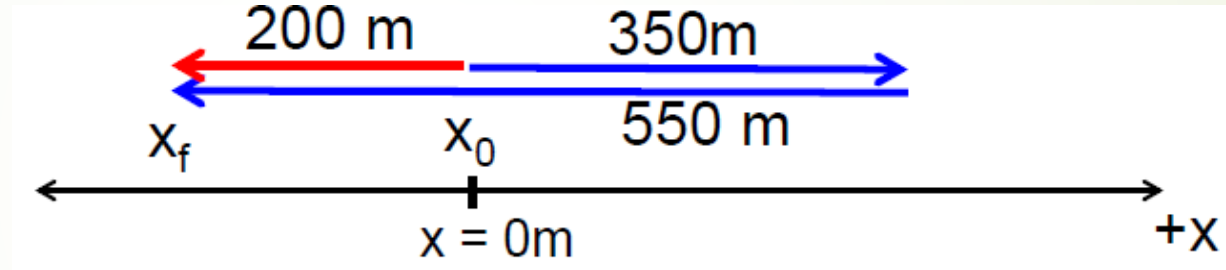
***** Displacement rarely equals Distance Traveled (only when traveling in the exact same direction the whole time)**



Origin
 $x=0, y=0$ or,
 $(x,y) = (0,0)$

Displacement vs. Distance

A cyclist rides 350 m due East, then turns around and rides again 550 m due West. Find the displacement and the distance traveled by the cyclist?



Displacement:

$$\Delta x = x_f - x_0 = -200\text{ m} - 0\text{ m} = -200\text{ m}$$

(200 m to the left of the origin)

Distance Traveled:

$$350\text{ m} + 550\text{ m} = 900\text{ m}$$

Speed

Average speed is the distance traveled divided by the time ($t_f - t_0$) required to cover the distance. Average value of v , is written as \bar{v} (v with a bar over it).

$$\text{Speed, } v = \frac{\text{distance}}{\text{elapsed time}}$$

SI units for speed: meters per second (m/s)

If the cyclist rides 900 meters in 5 mins his speed is 3 m/s (this is the average speed).

**Speed at any instant in time is termed *instantaneous* speed

Velocity

If we know the speed and the direction of motion we know the velocity

Constant speed - steady speed

Constant velocity

- steady speed and same direction
(same direction – straight line)

Average velocity is the displacement divided by the elapsed time.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_0}{t_f - t_0} \rightarrow x_f - x_0 = \bar{v} \Delta t$$

Instantaneous velocity – velocity at that instant (shrink Δt to very small)

Constant motion does not require a force

*** **Velocity** is a vector. You must give a speed (number plus units) and a direction (north or east, or + or -).

Scalars and Vectors

A scalar quantity specifies magnitude only; that is, value. **Speed** is one example of a scalar quantity.

List another scalar quantity and explain.

A vector quantity has magnitude + direction. **Velocity** is one example of a vector quantity.

List another vector quantity and explain.

An arrow is used to designate a vector with length indicating relative magnitude.

If \longrightarrow = 5 m/s N

Then \longrightarrow = 10 m/s N

And \longleftarrow = -5 m/s or 5 m/s S



Scalars and Vectors

A **scalar quantity** has only **magnitude**.

A **vector quantity** has both **magnitude** and **direction**.

Scalar Quantities

length, area, volume
speed
mass, density
pressure
temperature
energy, entropy
work, power



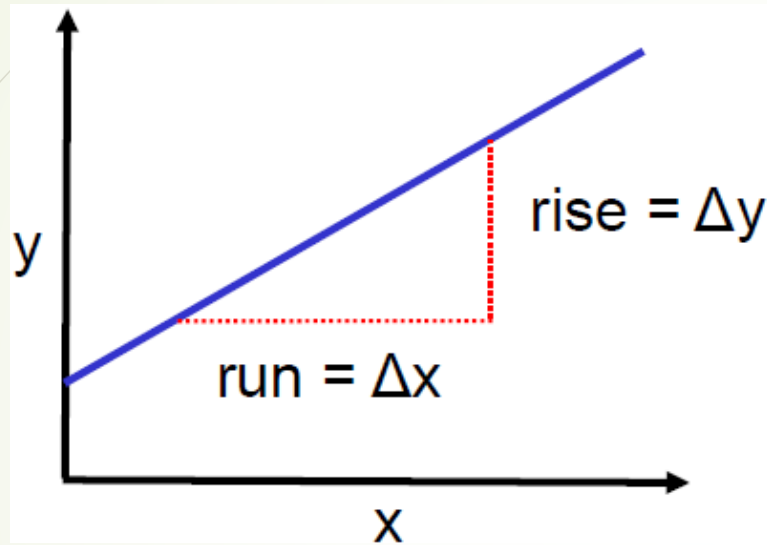
Vector Quantities

displacement
velocity
acceleration
momentum
force
lift, drag, thrust
weight



Graphical representation of motion

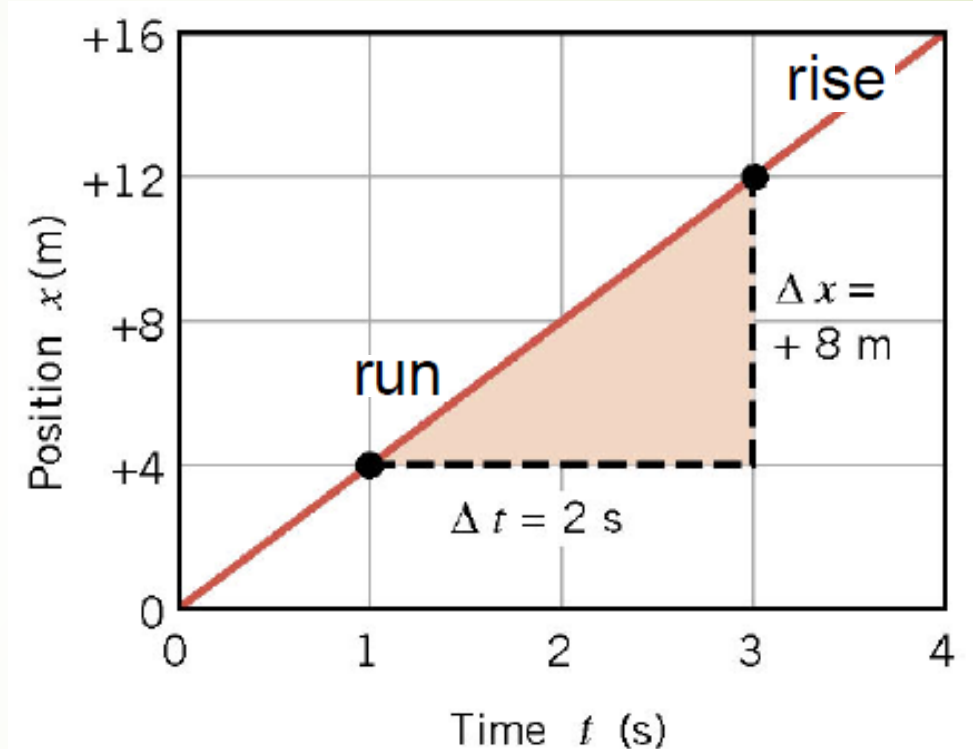
➤ Slope of a function on a graph



- Slope = rise/run = $\Delta y / \Delta x$
- Up to the right is a positive slope.
- Down to the right is negative slope.

Velocity = slope of line
(units read from the axis labels)
straight line = uniform motion

Graph of position vs. time.



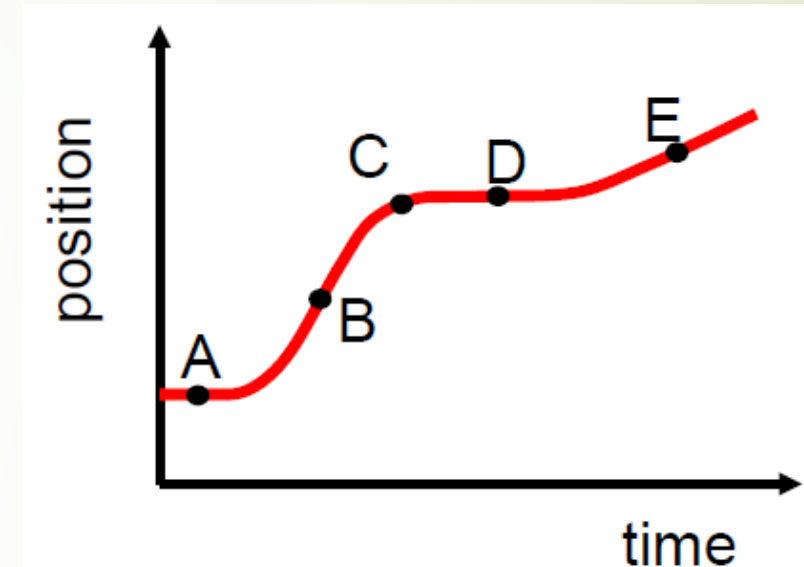
$$\text{Slope} = \frac{\Delta x}{\Delta t} = \frac{+8 \text{ m}}{2 \text{ s}} = +4 \text{ m/s}$$

The same slope at all times.
This means constant velocity!

Graphical representation of motion

This position versus time graph represents the position of a person as a function of time. At what point or points is the person standing still?

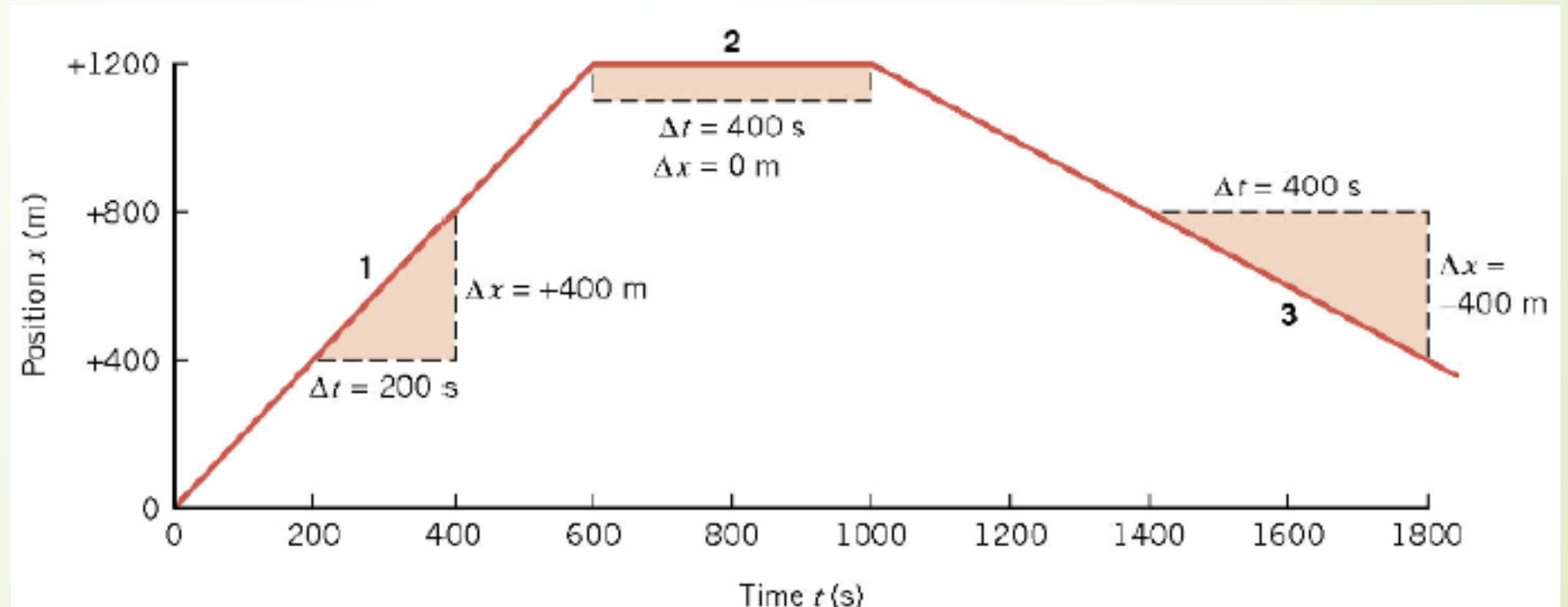
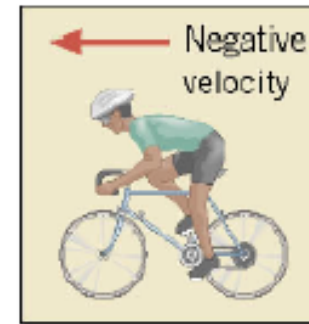
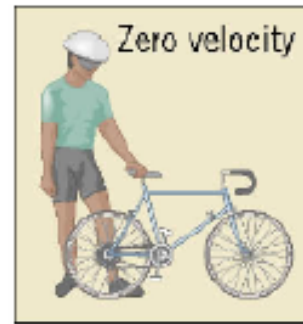
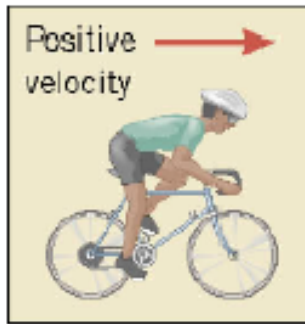
- a) A
- b) B
- c) C
- d) D
- e) A and D
- f) B and E



Try to replicate this graph using the following PhET: [Moving Man](#)

Graphical representation of motion

This position versus time graph represents the position of a cyclist as a function of time. What is the slope (velocity) of each of these lines?

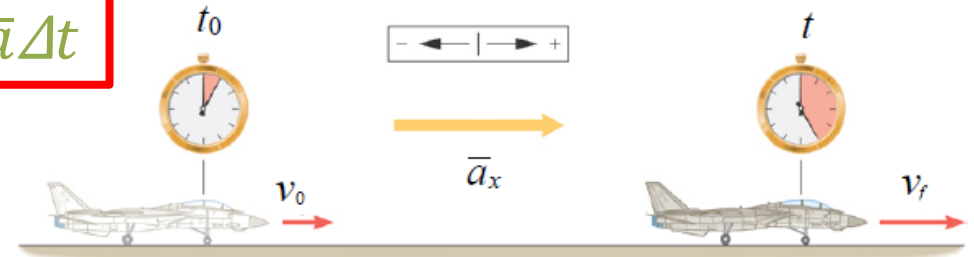


Acceleration

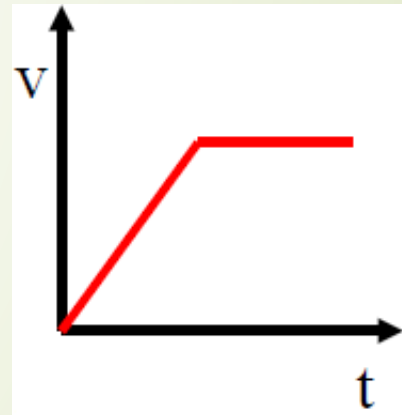
Acceleration is the rate of change of velocity. *Acceleration* is a vector!

Average acceleration is the change in the velocity over the elapsed time.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0} \longrightarrow \boxed{v_f - v_0 = \bar{a}\Delta t}$$



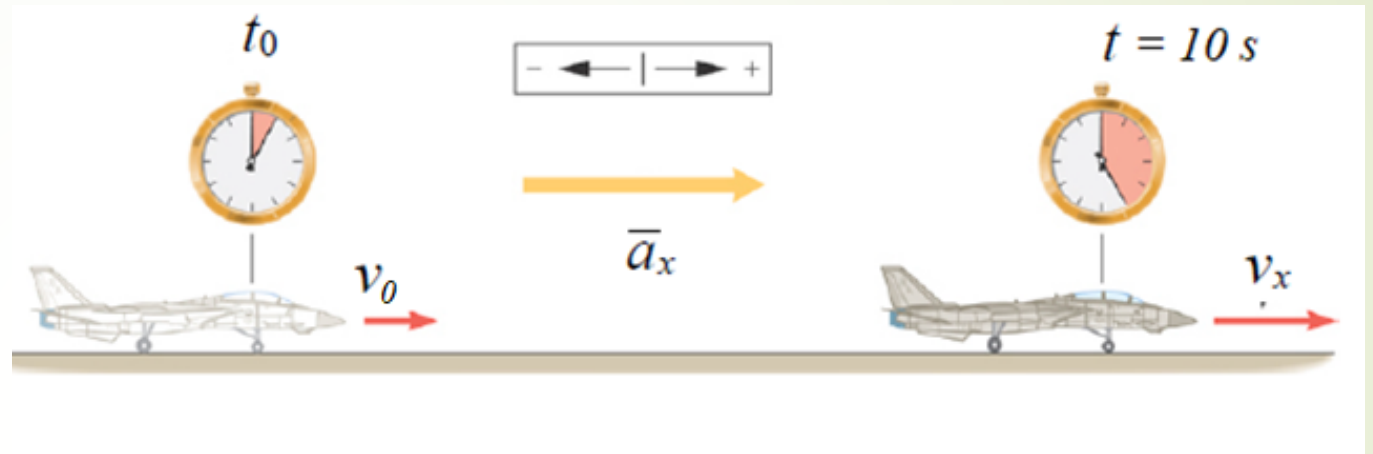
- **Change of velocity** could result from change of speed, change of direction or change of both speed and direction.
- If net force constant, acceleration is constant (uniform motion or constant velocity)
- **S.I. unit for acceleration** – m/s/s or m/s²



*** **Acceleration** would be *slope* on v vs. t graph (changing velocity) and also result in a change in x vs. t graph

Acceleration

Example: Suppose you are taking off and you change the velocity steadily from 40 km/h to 45 km/h in 10 seconds. In next 10 seconds you increase it to 50km/h, and soon.



You change velocity by 5km/h in every 10 seconds.

$$\begin{aligned}\text{Acceleration} &= \frac{\text{change of velocity}}{\text{time interval}} \\ &= \frac{45 \text{ km/h} - 40 \text{ km/h}}{10 \text{ s}} \\ &= \frac{5 \text{ km/h}}{10 \text{ s}} = 0.5 \text{ km/h/s}\end{aligned}$$

**This calculation of the average acceleration works even if the acceleration is not constant throughout the motion.*

Equations of Kinematics for Constant Acceleration

If constant net force \longrightarrow Then constant acceleration

Five relevant kinematic variables and equations to describe such motion:

1. displacement, Δx
2. acceleration (constant), a *no average bar is needed*
3. final velocity (at time t), v_f
4. initial velocity, v_0
5. elapsed time, Δt

$$x = x_0 + \bar{v}t$$

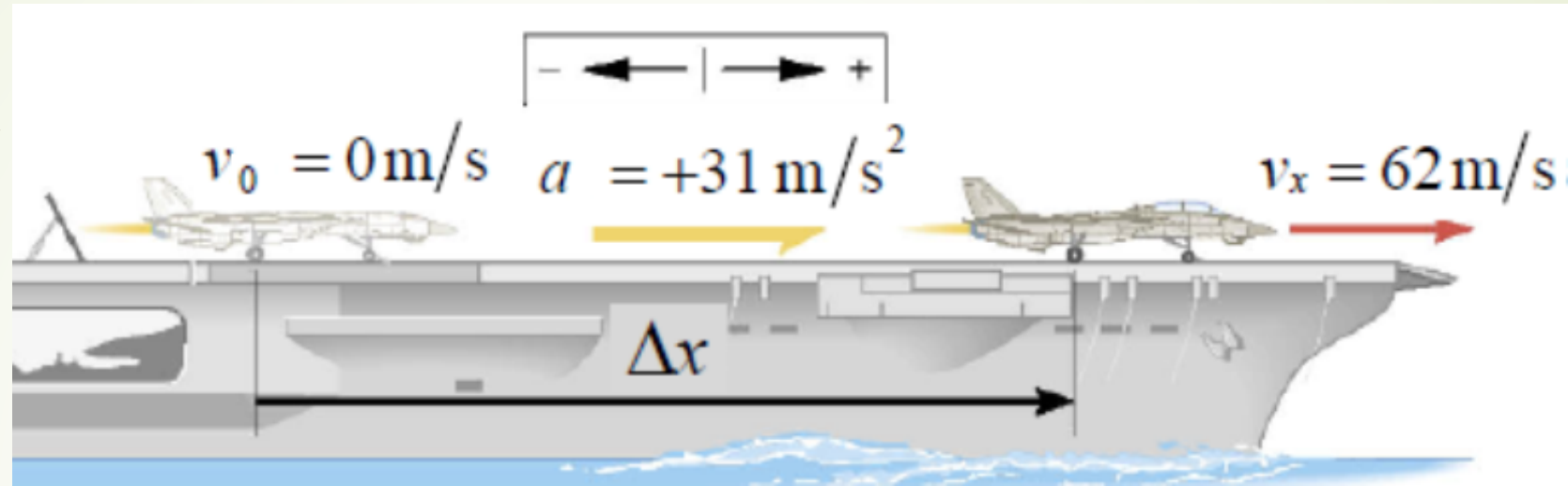
$$\bar{v} = \frac{1}{2}(v_0 + v)$$

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

Kinematics of Constant Acceleration



Example: Find the displacement, while catapulting a jet from rest to 62m/s with an acceleration of $+31 \text{ m/s}^2$.

$$\Delta x = \frac{v_x^2 - v_0^2}{2a} = \frac{(62 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(31 \text{ m/s}^2)} = +62 \text{ m}$$

Freely Falling Bodies

For vertical motion, we will replace the x label with y in all kinematic equations, and use **upward as positive**.

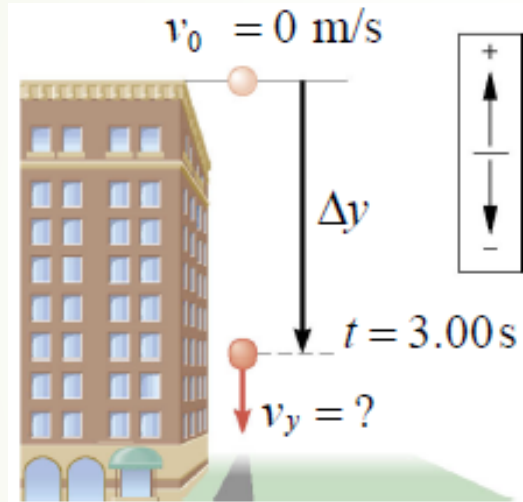
It is found that, when objects are in free fall and thus not affected by air resistance (drag), all bodies at the same location above the Earth fall vertically with the same acceleration. Also, if the distance of the fall is small compared to the radius of the Earth, then the acceleration remains essentially constant throughout the descent.

Such idealized motion is called *free-fall* and the acceleration of a freely falling body is called the *acceleration due to gravity*, and the acceleration is **downward or negative**.

$$a = -g = -9.81\text{m/s}^2 \quad \text{or} \quad -32.2\text{ft/s}^2$$

Freely Falling Bodies

Example: A coin from the top of a tall building is dropped from rest. After 3.00s of free fall, what is the vertical displacement, Δy of the coin?



Δy	$a = -g$	v_y	v_0	t
?	-9.80 m/s^2		0 m/s	3.00 s

$$\begin{aligned}\Delta y &= v_0 t + \frac{1}{2} a t^2 \\ &= (0 \text{ m/s})(3.00 \text{ s}) + \frac{1}{2} (-9.80 \text{ m/s}^2)(3.00 \text{ s})^2 \\ &= -44.1 \text{ m}\end{aligned}$$

Freely Falling Bodies

Example: An object is dropped from the roof of Building A. It hits the ground after 3 seconds. The same object is dropped from Building B and takes 6 seconds to hit the ground. The height of building B is ___ times the height of Building A. (Ignore air resistance.)

- A. $\frac{1}{4}$
- B. $\frac{1}{2}$
- C. 4
- D. 1

Solution:

$$\begin{aligned}y &= y_0 + v_0 t + \frac{1}{2}at^2 \\ &= 0 \text{ m} + (0 \text{ m/s}) * t + \frac{1}{2}at^2 \\ &= \frac{1}{2}at^2\end{aligned}$$

So, if you double t , then y becomes four times