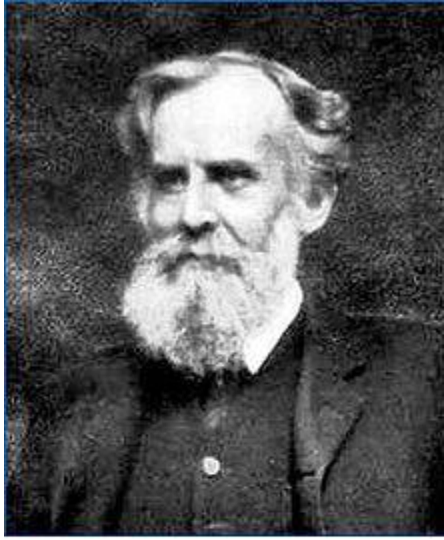
Venn Diagrams and Categorical Syllogisms

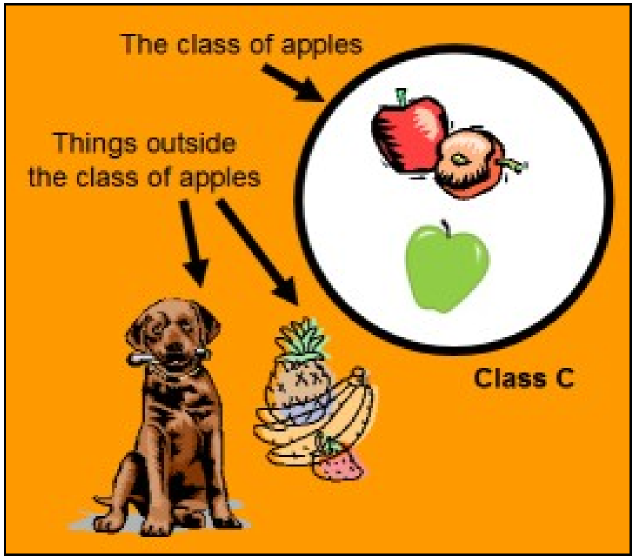
Unit 5

John Venn



- 1834 – 1923
- English logician and philosopher noted for introducing the Venn diagram
- Used in set theory, probability, logic, statistics, and computer science

Classes of things represented in circles:



Let us start with the concept of a class. A *class* or a *set* is simply a collection of objects. These objects are called *members* of the set. A class is defined by its members.

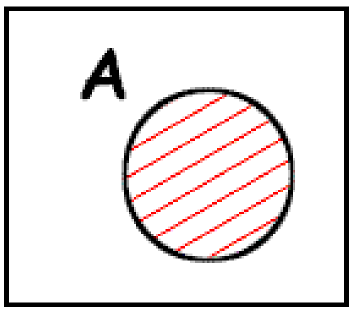
So for example, we might define a class C as the class of apples. In that case, every apple in the world is a member of C, and anything that is not an apple is not a member of C. If something is not a member of a class, we can also say that the object is *outside the class*.

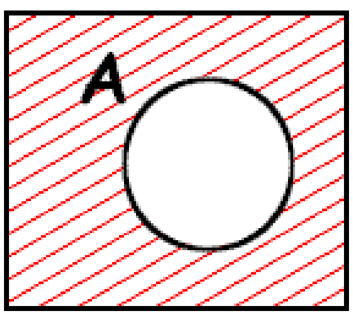
Note that a class can be empty. The class of men over 5 meters tall is presumably empty since nobody is that tall. The class of plane figures that are both round and square is also empty since nothing can be both round and square.

A class can also be infinite, containing an infinite number of objects. The class of even number is an example. It has infinitely many members, including 2, 4, 6, 8, and so on.

Let us now see how Venn diagrams are used to represent classes.

Shading means the class or set of items is empty – there are none that fit that description:

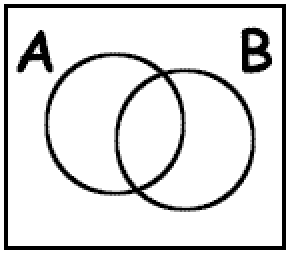




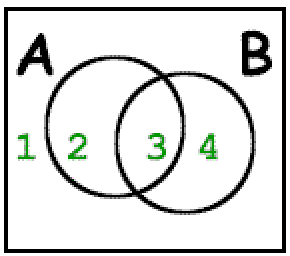
Let us now consider what shading means:

- To indicate that a class is empty, we shade the circle representing that class. So the diagram on the left means that class A is empty.
- In general, shading an area means that the class represented by the area is empty. So the second diagram on the left represents a situation where there isn't anything which is not a member of class A.
- However, even though shading indicates emptiness, a region that is not shaded does not necessarily indicate a non-empty class. As we shall see in the next tutorial, we use a tick to indicate existence. In the second diagram on the left, the region marked A is not shaded. This does not imply that there are things which exist which are members of A. If the area is blank, this means that *we do not have any information as to whether there is anything there*.

Comparing two different classes:



Now let us consider a slightly more complicated diagram where we have two *intersecting* circles. The one on the left represents a class A. The other one represents a classs B.



Let us label the different bounded regions. Then:

- Region 1 represents objects which are neither A nor B. This is because this area is outside both the A circle and the B circle.
- Region 2 represents objects which have property A but which do not have property B.
- Region 3 represents objects which have both property A and property B.
- Region 4 represents objects which have property B but not A.

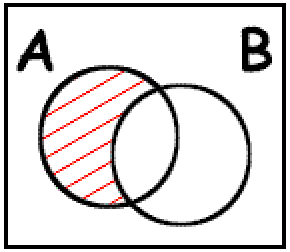
So for example, suppose B is the class of sweet things. In that case what does region three represent?
Answer

The class of sweet apples!

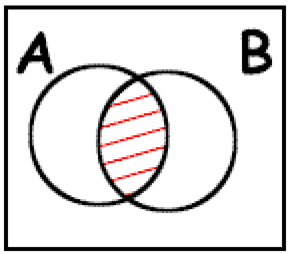
Furthermore, which region represents the class that contains sour lemons? Answer

Region #1

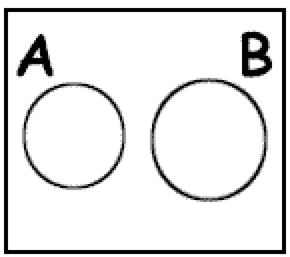
All are, “every” vs. No are, “none”, “nothing”



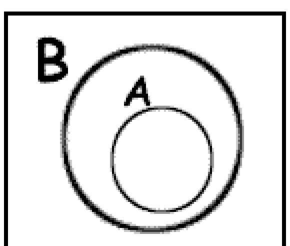
Continuing with our diagram, suppose we now shade region 2. This means that the class of things which are A but not B is empty. In other words, **every A is a B**. (It might be useful to note that this is equivalent to saying that if anything is an A, it is also a B.) This is an important point to remember. Whenever you want to represent "every A is B", shade the area within the A circle that is outside the B circle.



What if we shade the middle region where A and B overlaps? This is the region representing things which are both A and B. So shading indicates that **nothing is both A and B**. If you think about it carefully, you will see that "Nothing is both A and B" says the same thing as "No A is a B" and "No B is an A". Make sure that you understand why these claims are logically equivalent!



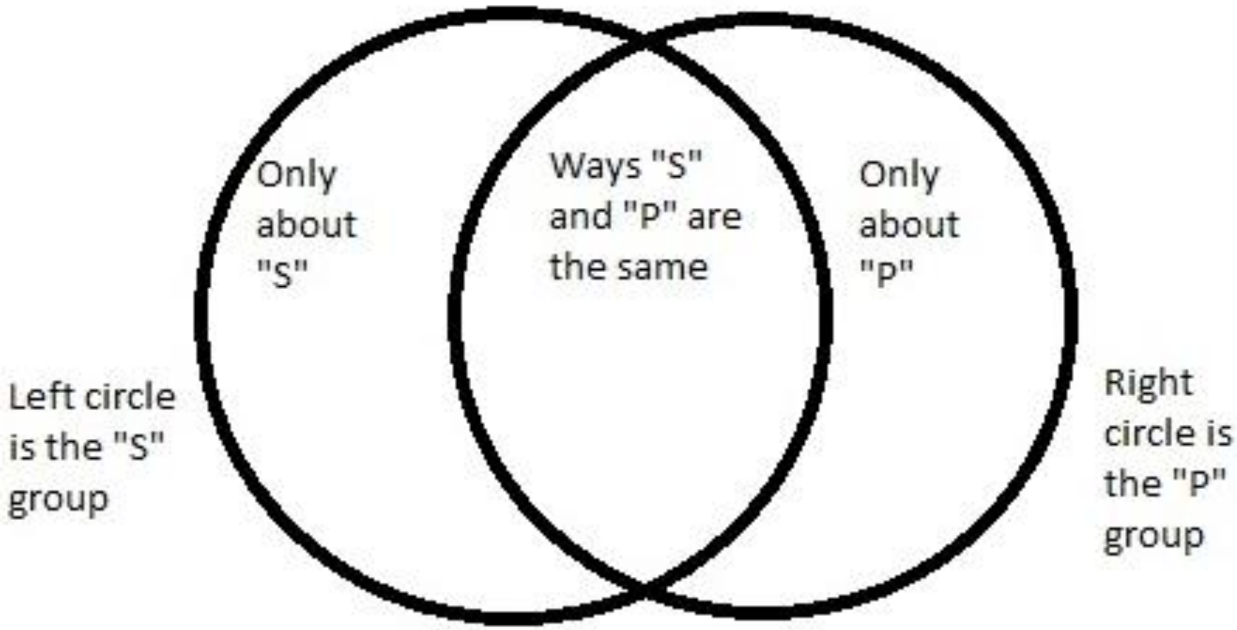
Incidentally, we could have represented the same information by using two non-overlapping circles instead.



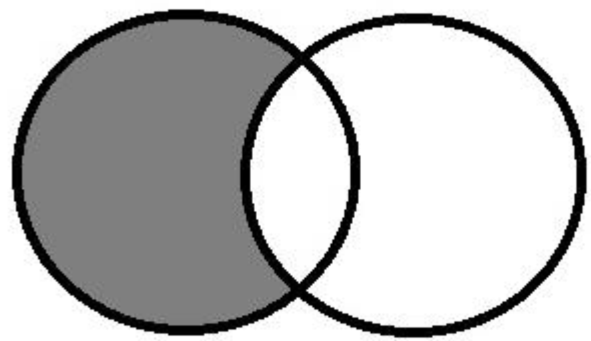
What about the diagram on the left? What do you think it represents? Answer

Every A is B. Because the A circle is inside the B circle, every member of A is also a member of B. But there might be things which are B but not A.

Representing subjects and predicates visually:



All S are P.



Universal Categorical Statement
All members of the **subject** class are also members of the **predicate** class.

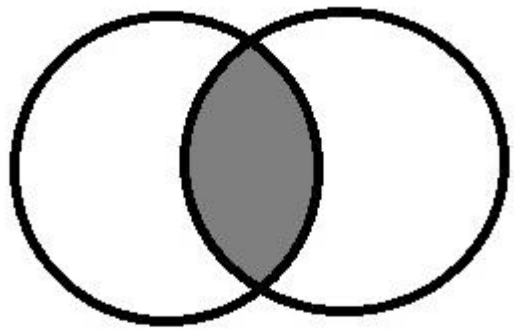
All sharks are predators. (The circle on the left represents sharks, the circle on the left is predators.)

All apples are sweet.

We’ve shaded out the area where the non-predator sharks would have been, and where the non-sweet apples would have been.

This is known as an “A” statement.

No S are P.



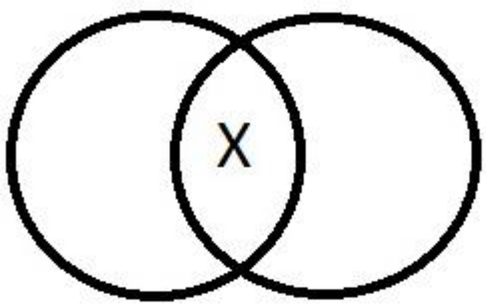
Universal Categorical Statement
No members of the **subject** class are also members of the **predicate** class.

No salmon are porpoises.

We shaded the area where a “salmon-porpoise” would have been, because there cannot be anything existing in that area.

This is known as an “E” statement.

Some S are P.



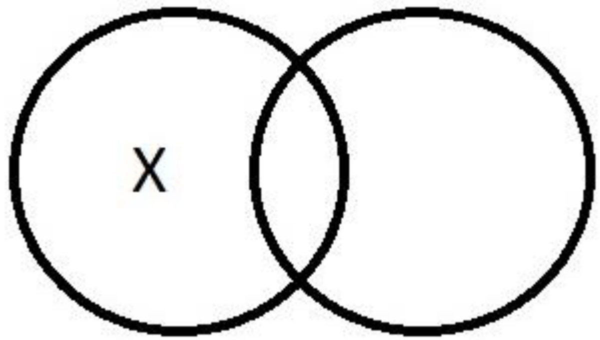
Particular Categorical Statement
Some members of the subject class are also members of the predicate class.

Some sailors are privates.

The X is placed where the sailors who are also privates would exist.

This is known as the “I” statement.

Some S are not P.



Particular Categorical Statement
Some members of the subject class are not members of the predicate class.

Some scholars are not philosophers.

This is known as the “O” form statement.

A, E, I, O

- There are four basic categorical statement types, named A, E, I, and O.
- The *quantity* of a categorical proposition is either universal, or particular.
- The *quality* of a categorical proposition is either affirmative, or negative.

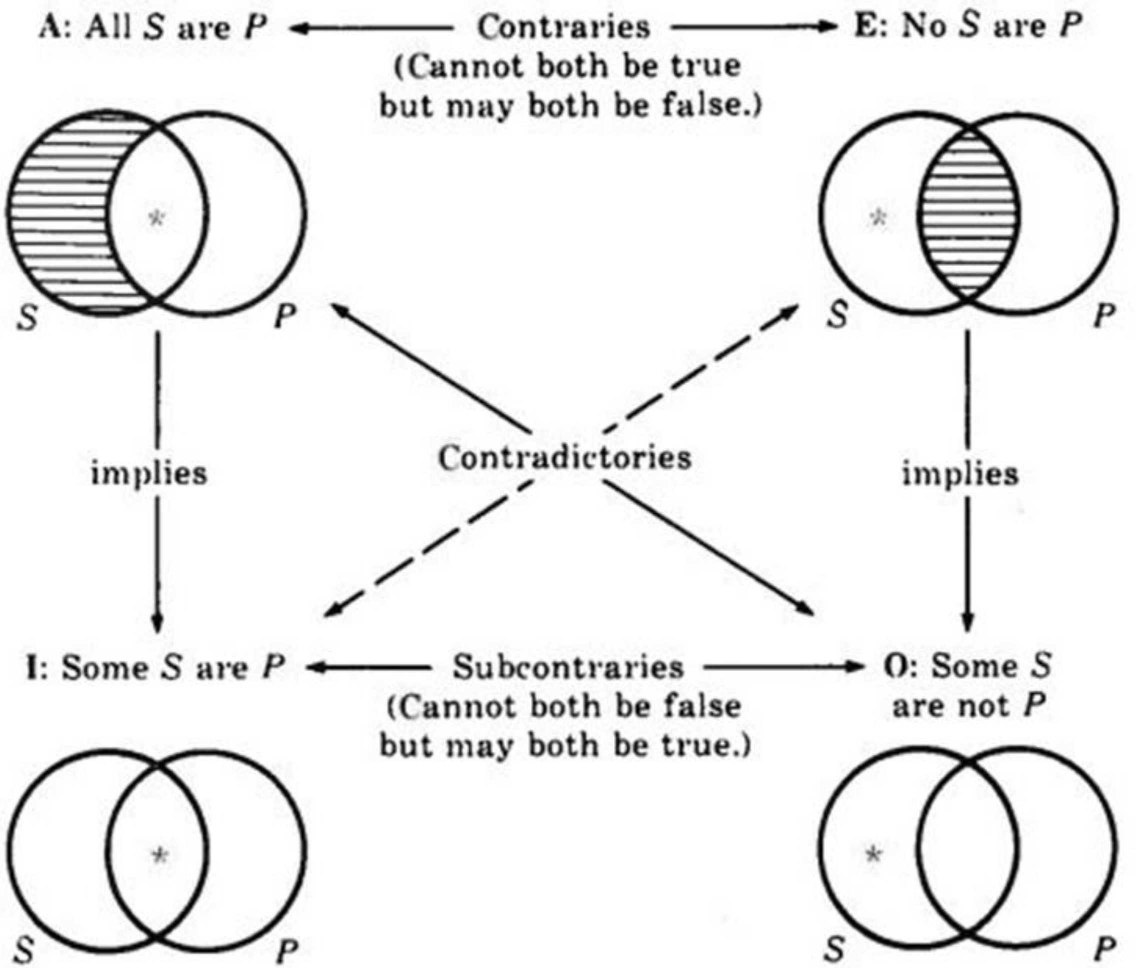
A, E, I, O and their quantity and quality

- A is the universal affirmative statement.
- E is the universal negative statement.
- I is the particular affirmative statement.
- O is the particular negative statement.

The meaning of “some”

- In categorical logic “some” means “at least one”, which is considered its minimal meaning.
- “Some” is the standard form *particular* quantifier.
- So, for example, the I statement, “Some buses used by the city are roadworthy machines”, would mean there’s at least one bus used by the city (but not necessarily any more than that one) that is a roadworthy machine.

The Square of Opposition: How the four types of categorical statements relate to each other



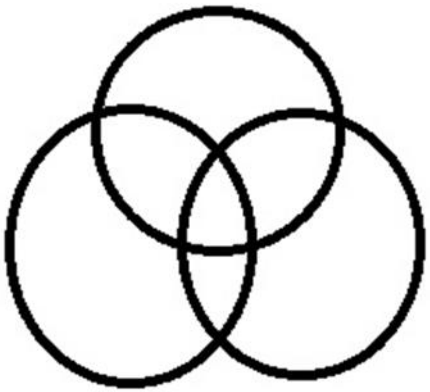
Practice:

Which diagram goes with each of the following categorical propositions? Give the letter type, and draw the Venn diagram.

- 1. No Catholics are Protestants.
- 2. Some cats are Persians.
- 3. Some dogs are not hunters.
- 4. All Georgians are Americans.
- 5. No fish are bipeds.

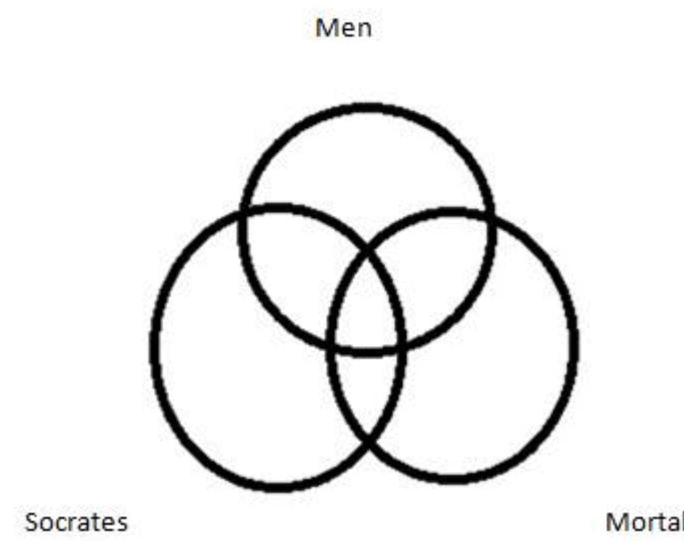
Categorical Syllogisms

- Made up of three statements, giving three specific classes of things and how they relate
- http://philosophy.hku.hk/think/venn/tute3.php
- The webpage above is interactive, and shows how the three statements can be placed on a Venn Diagram made of three circles.

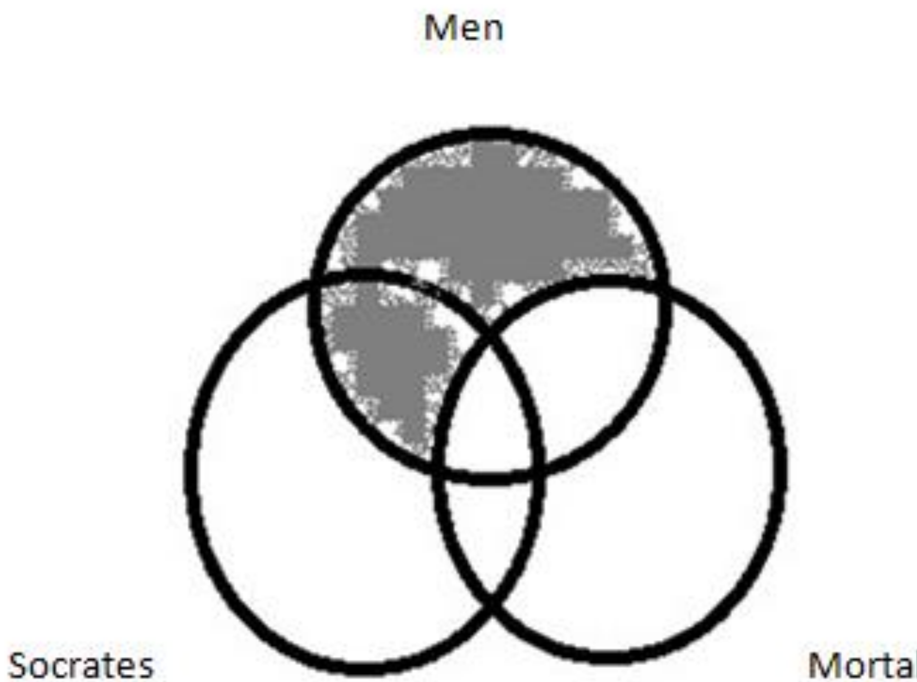


Imagine how to shade in:

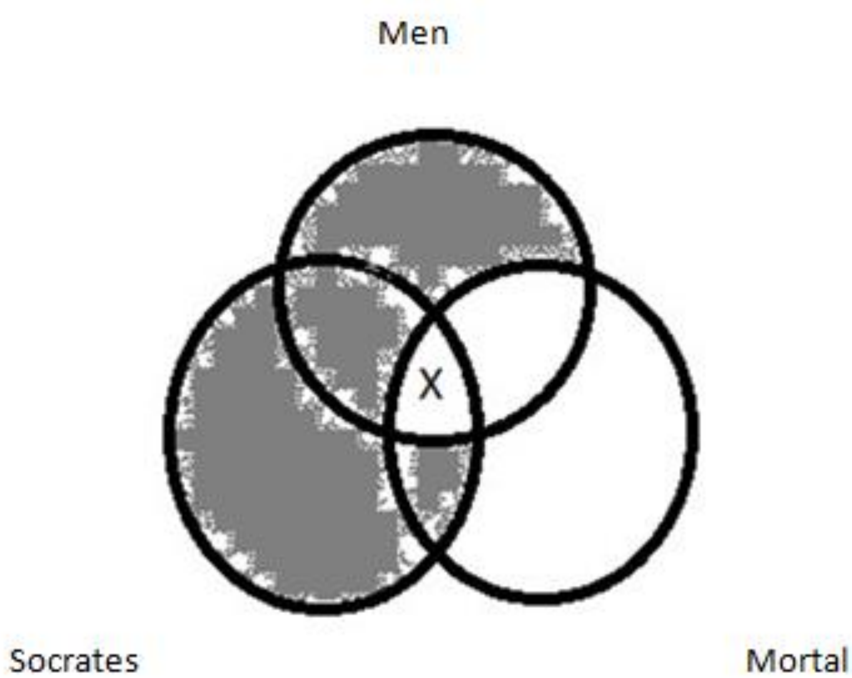
- All men are mortal.
- Socrates is a man.
- Therefore, Socrates is mortal.



First shade in All men are mortal:

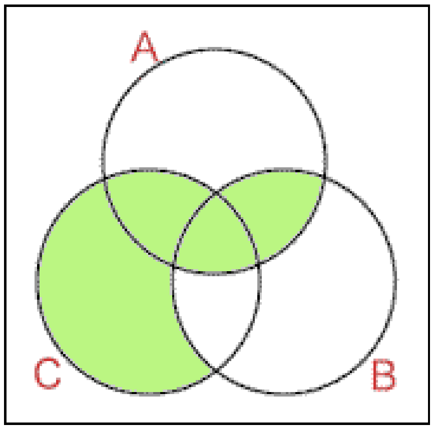


Socrates is a Man
And check to see if there is a space where Socrates can be mortal as well – there is, where the X appears below:



This is considered “valid” because the Venn diagram shows that the conclusion is necessarily true: there is no space left for a Socrates who is *non*-mortal!

Checking what is true on a Venn diagram:

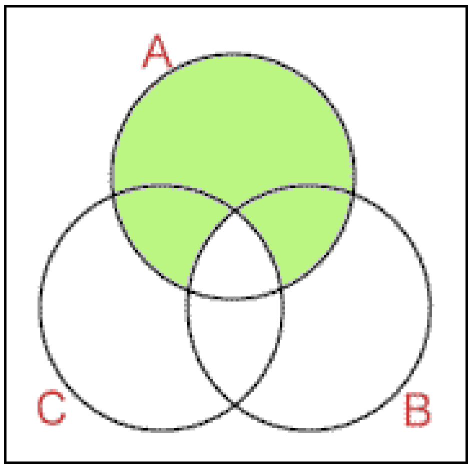


Q1 Is the statement "Every C is a B" true according to this diagram?

Answer

Correct.

Checking what is true on a Venn diagram:



Q3 Is the diagram consistent with the statement "Something is A"?

Answer

Yes. There is still one unshaded region within the A cricle.

Venn Diagram checker

- http://philosophy.hku.hk/think/venn/venn-program.php
- Use this link to double check how you represent statements on Venn diagrams
- The page immediately after it explains how to use “ticks” on a Venn diagram to check Categorical Syllogism arguments for validity

Mood (cont.)

- Example:
- *All dictators are tyrants.*
- *All czars are dictators.*
- *Therefore, all czars are tyrants.*

- The mood of this example is *AAA*.

A Question For You

- Selecting from the list below, what is the mood of this syllogism?

- *No candidates are witnesses.*
- *Some lawyers are witnesses.*
- *So, some lawyers are not candidates.*

- A. OIE
- B. IOE
- C. EIO
- D. AOI

Answer

- Selecting from the list below, what is the mood of this syllogism?

- *No candidates are witnesses.*
- *Some lawyers are witnesses.*
- *So, some lawyers are not candidates.*

- A. OIE
- B. IOE
- C. EIO **(Correct)**
- D. AOI

Figure

- The figure of a syllogism is determined by the position of the middle term.
- Example:
- *All dictators are tyrants.*
- *All czars are dictators.*
- *Therefore, all czars are tyrants.*

- In this example the middle term is “dictators”.

Substituting the letters for the terms

(M) dictators- (P) tyrants
<u>(S) czars- (M) dictators</u>
(S) czars- (P) tyrants

This gives us:

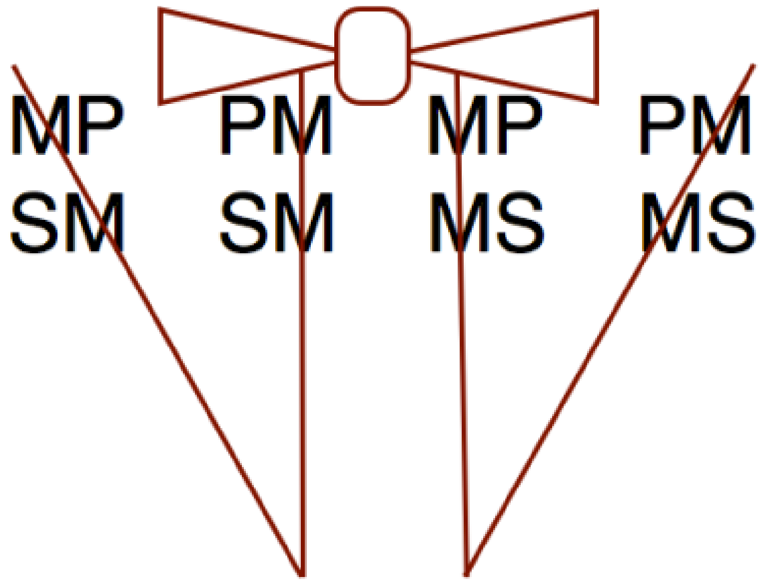
$$\frac{\begin{array}{c} M\ P \\ S\ M \end{array}}{S\ P}$$

S refers to the subject term of the conclusion.
P refers to the predicate term of the conclusion.
M is the middle term, it only appears in the premises.

The 4 Figures:

M P	P M	M P	P M
S M	S M	M S	M S
S P	S P	S P	S P
1^{ST}figure	2^{ND}figure	3^{rd} figure	4^{th} figure

Figures represented as a bow tie and collar on the M (middle terms)



M	P	P	M	M	P	P	M
S	M	S	M	M	S	M	S
S	P	S	P	S	P	S	P

Figure: 1 2 3 4