19 ELECTRIC POTENTIAL AND ELECTRIC FIELD

Figure 19.1 Automated external defibrillator unit (AED) (credit: U.S. Defense Department photo/Tech. Sgt. Suzanne M. Day)

Learning Objectives

19.1. Electric Potential Energy: Potential Difference
- Define electric potential and electric potential energy.
- Describe the relationship between potential difference and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.
- Determine electric potential energy given potential difference and amount of charge.

19.2. Electric Potential in a Uniform Electric Field
- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

19.3. Electrical Potential Due to a Point Charge
- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

19.4. Equipotential Lines
- Explain equipotential lines and equipotential surfaces.
- Describe the action of grounding an electrical appliance.
- Compare electric field and equipotential lines.

19.5. Capacitors and Dielectrics
- Describe the action of a capacitor and define capacitance.
- Explain parallel plate capacitors and their capacitances.
- Discuss the process of increasing the capacitance of a dielectric.
- Determine capacitance given charge and voltage.

19.6. Capacitors in Series and Parallel
- Derive expressions for total capacitance in series and in parallel.
- Identify series and parallel parts in the combination of connection of capacitors.
- Calculate the effective capacitance in series and parallel given individual capacitances.

19.7. Energy Stored in Capacitors
- List some uses of capacitors.
- Express in equation form the energy stored in a capacitor.
- Explain the function of a defibrillator.

Introduction to Electric Potential and Electric Energy

In **Electric Charge and Electric Field**, we just scratched the surface (or at least rubbed it) of electrical phenomena. Two of the most familiar aspects of electricity are its energy and *voltage*. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at molecular levels, *ions* cross cell membranes and transfer information. We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. In this chapter, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

19.1 Electric Potential Energy: Potential Difference

When a free positive charge q is accelerated by an electric field, such as shown in **Figure 19.2**, it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.

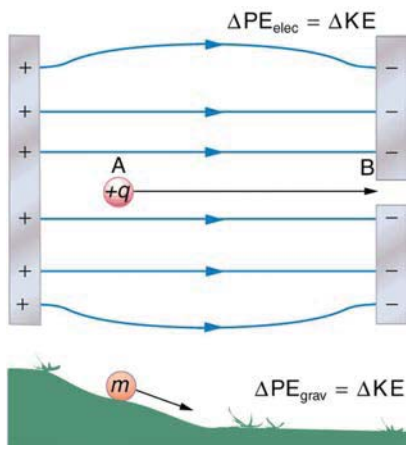

Figure 19.2 A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force is conservative, we can write $W = -\Delta\text{PE}$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, ΔPE , is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE . There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Potential Energy

$W = -\Delta\text{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative $\Delta\text{PE}.$ There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since $W = Fd\cos\theta$ and the direction and magnitude of F can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since $F = qE$, the work, and hence ΔPE , is proportional to the test charge $q.$ To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

$$V = \frac{\text{PE}}{q}. \tag{19.1}$$

Electric Potential

This is the electric potential energy per unit charge.

$$V = \frac{\text{PE}}{q} \tag{19.2}$$

Since PE is proportional to q, the dependence on q cancels. Thus V does not depend on q. The change in potential energy ΔPE is crucial, and so we are concerned with the difference in potential or potential difference ΔV between two points, where

$$\Delta V = V_{\text{B}} - V_{\text{A}} = \frac{\Delta\text{PE}}{q}. \tag{19.3}$$

The **potential difference** between points A and B, $V_{\text{B}} - V_{\text{A}}$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\text{ V} = 1\,\frac{\text{J}}{\text{C}} \tag{19.4}$$

Potential Difference

The potential difference between points A and B, V_{B} - V_{A}, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

$$1\text{ V} = 1\,\frac{\text{J}}{\text{C}} \tag{19.5}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta\text{PE}}{q} \text{ and } \Delta\text{PE} = q\Delta V. \tag{19.6}$$

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

$$\Delta V = \frac{\Delta\text{PE}}{q} \text{ and } \Delta\text{PE} = q\Delta V. \tag{19.7}$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta\text{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Example 19.1 Calculating Energy

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta\text{PE} = q\Delta V$.

So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000\text{ C}$ and $\Delta V = 12.0\text{ V}$. The total energy delivered by the motorcycle battery is

$$\begin{aligned} \Delta\text{PE}_{\text{cycle}} &= (5000\text{ C})(12.0\text{ V}) \\ &= (5000\text{ C})(12.0\text{ J/C}) \\ &= 6.00\times10^{4}\text{ J}. \end{aligned} \tag{19.8}$$

Similarly, for the car battery, $q = 60{,}000\text{ C}$ and

$$\begin{aligned} \Delta\text{PE}_{\text{car}} &= (60{,}000\text{ C})(12.0\text{ V}) \\ &= 7.20\times10^{5}\text{ J}. \end{aligned} \tag{19.9}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in **Figure 19.3**. The change in potential is $\Delta V = V_{\text{B}} - V_{\text{A}} = +12\text{ V}$ and the charge q is negative, so that $\Delta\text{PE} = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.

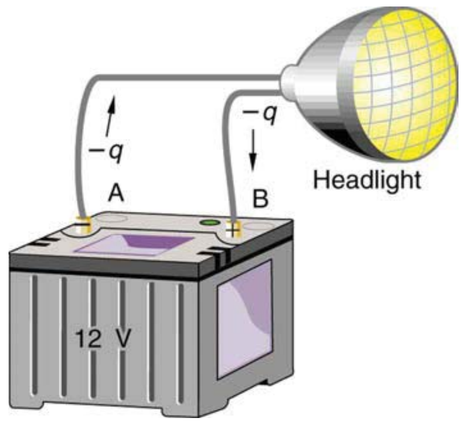

Figure 19.3 A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

Example 19.2 How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta\text{PE} = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta\text{PE} = -30.0\text{ J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\text{ V}$.

Solution

To find the charge q moved, we solve the equation $\Delta\text{PE} = q\Delta V$:

$$q = \frac{\Delta\text{PE}}{\Delta V}. \tag{19.10}$$

Entering the values for ΔPE and ΔV, we get

$$q = \frac{-30.0\text{ J}}{+12.0\text{ V}} = \frac{-30.0\text{ J}}{+12.0\text{ J/C}} = -2.50\text{ C}. \tag{19.11}$$

The number of electrons n_{e} is the total charge divided by the charge per electron. That is,

$$n_{\text{e}} = \frac{-2.50\text{ C}}{-1.60\times10^{-19}\text{ C/e}^{-}} = 1.56\times10^{19}\text{ electrons.} \tag{19.12}$$

Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. **Figure 19.4** shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta \text{PE} = q\Delta V,$ we can think of the joule as a coulomb-volt.

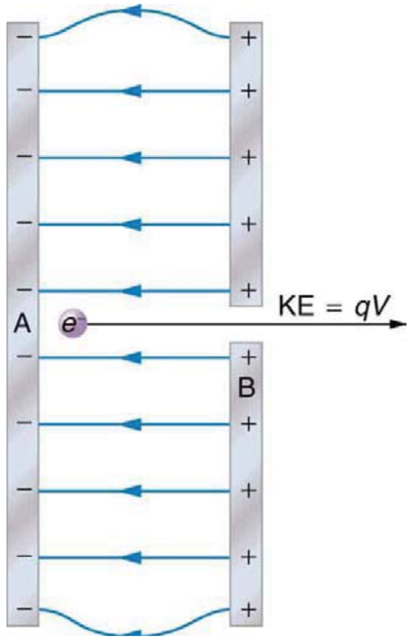

Figure 19.4 A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1 \text{ eV} &= \left(1.60\times10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60\times10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60\times10^{-19}\text{ J.} \end{aligned} \tag{19.13}$$

Electron Volt

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An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example, calories for food energy, kilowatt-hours for electrical energy, and therms for natural gas energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules ($30,000\text{ eV} \div 5\text{ eV per molecule} = 6000\text{ molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $\text{KE} + \text{PE} = \text{constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

$$\text{KE} + \text{PE} = \text{constant} \tag{19.15}$$

or

$$\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f} , \tag{19.16}$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example 19.3 Electrical Potential Energy Converted to Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $\text{KE}_\text{i} = 0, \text{KE}_\text{f} = ½mv^2, \text{PE}_\text{i} = qV, \text{and } \text{PE}_\text{f} = 0.$

Solution

Conservation of energy states that

$$\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f}. \tag{19.17}$$

Entering the forms identified above, we obtain

$$qV = \frac{mv^2}{2}. \tag{19.18}$$

We solve this for v:

$$v = \sqrt{\frac{2qV}{m}}. \tag{19.19}$$

Entering values for $q, V, \text{and } m$ gives

$$\begin{aligned} v &= \sqrt{\frac{2\left(-1.60\times10^{-19}\text{ C}\right)(-100\text{ J/C})}{9.11\times10^{-31}\text{ kg}}} \\ &= 5.93\times10^{6}\text{ m/s.} \end{aligned} \tag{19.20}$$

Discussion

Note that both the charge and the initial voltage are negative, as in **Figure 19.4**. From the discussions in **Electric Charge and Electric Field**, we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken into account. That is why a low voltage is considered (accurately) in this example.

19.2 Electric Potential in a Uniform Electric Field

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field $\mathbf{E}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B. (See **Figure 19.5**.) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either ΔV or $\mathbf{E}$ can be used to describe any charge distribution. ΔV is most closely tied to energy, whereas $\mathbf{E}$ is most closely related to force. ΔV is a **scalar** quantity and has no direction, while $\mathbf{E}$ is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by E below.) The relationship between ΔV and $\mathbf{E}$ is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in **Electric Potential Energy: Potential Difference**, this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.

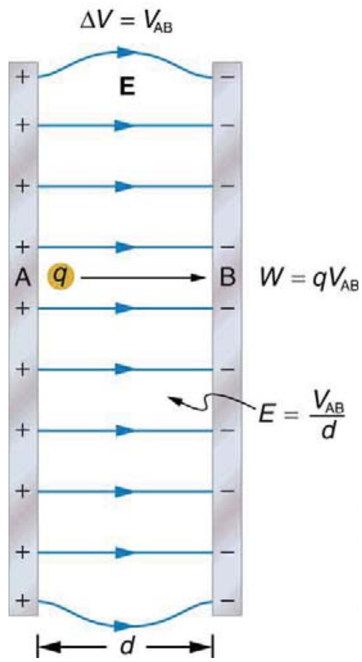

Figure 19.5 The relationship between V and E for parallel conducting plates is $E = V / d$. (Note that $\Delta V = V_{\text{AB}}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows: $-\Delta V = V_{\text{A}} - V_{\text{B}} = V_{\text{AB}}$. See the text for details.)

The work done by the electric field in **Figure 19.5** to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

$$W = -\Delta\text{PE} = -q\Delta V. \tag{19.21}$$

The potential difference between points A and B is

$$-\Delta V = -(V_{\text{B}} - V_{\text{A}}) = V_{\text{A}} - V_{\text{B}} = V_{\text{AB}}. \tag{19.22}$$

Entering this into the expression for work yields

$$W = qV_{\text{AB}}. \tag{19.23}$$

Work is $W = Fd\cos\theta$; here $\cos\theta = 1$, since the path is parallel to the field, and so $W = Fd$. Since $F = qE$, we see that $W = qEd$. Substituting this expression for work into the previous equation gives

$$qEd = qV_{\text{AB}}. \tag{19.24}$$

The charge cancels, and so the voltage between points A and B is seen to be

$$\left.\begin{array}{l} V_{\text{AB}} = Ed \\ E = \dfrac{V_{\text{AB}}}{d} \end{array}\right\}(\text{uniform } E\text{ - field on y}), \tag{19.25}$$

where d is the distance from A to B, or the distance between the plates in **Figure 19.5**. Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

$$1\text{ N}/\text{C} = 1\text{ V}/\text{m}. \tag{19.26}$$

Voltage between Points A and B

$$\left.\begin{array}{l} V_{\text{AB}} = Ed \\ E = \dfrac{V_{\text{AB}}}{d} \end{array}\right\}(\text{uniform } E\text{ - field on y}), \tag{19.27}$$

where d is the distance from A to B, or the distance between the plates.

Example 19.4 What Is the Highest Voltage Possible between Two Plates?

Dry air will support a maximum electric field strength of about $3.0\times10^{6}\text{ V/m}$. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. The equation $V_{AB} = Ed$ can thus be used to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

$$\mathrm{V}_{\mathrm{AB}} = Ed. \tag{19.28}$$

Entering the given values for E and d gives

$$V_{\mathrm{AB}} = (3.0\times10^{6}\ \mathrm{V/m})(0.025\ \mathrm{m}) = 7.5\times10^{4}\ \mathrm{V} \tag{19.29}$$

or

$$V_{\mathrm{AB}} = 75\ \mathrm{kV}. \tag{19.30}$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.

Figure 19.6 A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays). (credit: Daderot, Wikimedia Commons)

Example 19.5 Field and Force inside an Electron Gun

(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\ \mu\mathrm{C}$ charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{\mathrm{AB}}}{d}$. Once the electric field strength is known, the force on a charge is found using $\mathbf{F} = q\,\mathbf{E}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = q\,E$.

Solution for (a)

The expression for the magnitude of the electric field between two uniform metal plates is

$$E = \frac{V_{\mathrm{AB}}}{d}. \tag{19.31}$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

$$E = \frac{25.0\text{ kV}}{0.0400\text{ m}} = 6.25\times10^5\text{ V/m}. \tag{19.32}$$

Solution for (b)

The magnitude of the force on a charge in an electric field is obtained from the equation

$$F = qE. \tag{19.33}$$

Substituting known values gives

$$F = (0.500\times10^{-6}\text{ C})(6.25\times10^5\text{ V/m}) = 0.313\text{ N}. \tag{19.34}$$

Discussion

Note that the units are newtons, since $1\text{ V/m} = 1\text{ N/C}$. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\mathbf{E}$ and also in the direction of lower potential V. Furthermore, the magnitude of $\mathbf{E}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the greater the electric field. In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \tag{19.35}$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Relationship between Voltage and Electric Field

In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s}, \tag{19.36}$$

where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals and differential calculus must be employed to determine the electric field.

19.3 Electrical Potential Due to a Point Charge

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q, and noting the connection between work and potential $(W = -q\Delta V)$, it can be shown that the *electric potential V of a point charge* is

$$V = \frac{kQ}{r}\text{ (Point Charge)}, \tag{19.37}$$

where *k* is a constant equal to $9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2$.

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

$$V = \frac{kQ}{r}\text{ (Point Charge)}. \tag{19.38}$$

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas $\mathbf{E}$ for a point charge decreases with distance squared:

$$E = \frac{F}{q} = \frac{kQ}{r^2}. \tag{19.39}$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\mathbf{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\mathbf{E}$ is closely associated with force, a vector.

Example 19.6 What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a $-3.00\ \text{nC}$ static charge?

Strategy

As we have discussed in **Electric Charge and Electric Field**, charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation $V = kQ/r$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

$$\begin{aligned} V &= k\frac{Q}{r} \\ &= \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-3.00\times10^{-9}\ \text{C}}{5.00\times10^{-2}\ \text{m}}\right) \\ &= -539\ \text{V}. \end{aligned} \tag{19.40}$$

Discussion

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

Example 19.7 What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See **Figure 19.7**.) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)

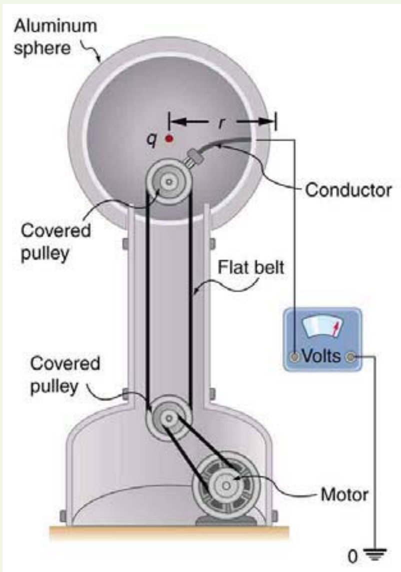

Figure 19.7 The voltage of this demonstration Van de Graaff generator is measured between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.) We can thus determine the excess charge using the equation

$$V = \frac{kQ}{r}. \tag{19.41}$$

Solution

Solving for Q and entering known values gives

$$\begin{aligned} Q &= \frac{rV}{k} \\ &= \frac{(0.125\ \text{m})\left(100\times10^{3}\ \text{V}\right)}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} \\ &= 1.39\times10^{-6}\ \text{C} = 1.39\ \mu\text{C}. \end{aligned} \tag{19.42}$$

Discussion

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in **Electric Potential Energy: Potential Difference**, this is analogous to taking sea level as $h=0$ when considering gravitational potential energy, $\mathrm{PE_g} = mgh$.

19.4 Equipotential Lines

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Of course, the two are related. Consider **Figure 19.8**, which shows an isolated positive point charge and its electric field lines. Electric field lines radiate out from a positive charge and terminate on negative charges. While we use blue arrows to represent the magnitude and direction of the electric field, we use green lines to represent places where the electric potential is constant. These are called **equipotential lines** in two dimensions, or *equipotential surfaces* in three dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true since the potential for a point charge is given by $V = kQ/r$ and, thus, has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of **Figure 19.8**. Since the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.

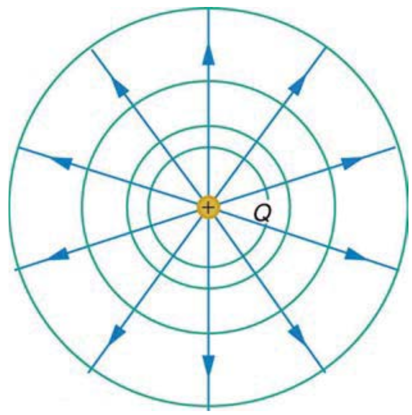

Figure 19.8 An isolated point charge Q with its electric field lines in blue and equipotential lines in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case.

It is important to note that *equipotential lines are always perpendicular to electric field lines.* No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus the work is

$$W = -\Delta\mathrm{PE} = -q\Delta V = 0. \tag{19.43}$$

Work is zero if force is perpendicular to motion. Force is in the same direction as $\mathbf{E}$, so that motion along an equipotential must be perpendicular to $\mathbf{E}$. More precisely, work is related to the electric field by

$$W = Fd\cos\theta = qEd\cos\theta = 0. \tag{19.44}$$

Note that in the above equation, E and F symbolize the magnitudes of the electric field strength and force, respectively. Neither q nor $\mathbf{E}$ nor d is zero, and so $\cos\theta$ must be 0, meaning θ must be 90°. In other words, motion along an equipotential is perpendicular to $\mathbf{E}$.

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations.* There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to the earth.

Grounding

A conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called grounding.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in **Figure 19.8** a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

Figure 19.9 shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in **Figure 19.10**(a), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in **Figure 19.10**(b).

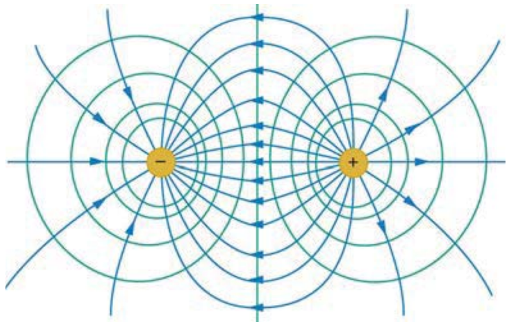

Figure 19.9 The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge.

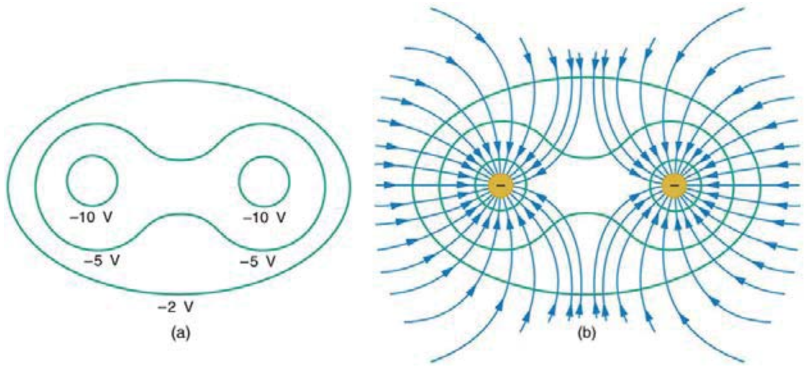

Figure 19.10 (a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges.

One of the most important cases is that of the familiar parallel conducting plates shown in **Figure 19.11**. Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.

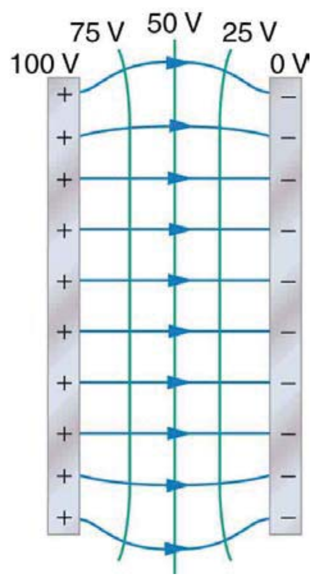

Figure 19.11 The electric field and equipotential lines between two metal plates.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart. More about the relationship between electric fields and the heart is discussed in **Energy Stored in Capacitors**.

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

PhET Interactive Simulation

Figure 19.12 Charges and Fields (http://cnx.org/content/m42331/1.3/charges-and-fields_en.jar)

19.5 Capacitors and Dielectrics

A **capacitor** is a device used to store electric charge. Capacitors have applications ranging from filtering static out of radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another, but not touching, such as those in **Figure 19.13**. (Most of the time an insulator is used between the two plates to provide separation—see the discussion on dielectrics below.) When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, $+Q$ and $-Q$, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance.

Capacitor

A capacitor is a device used to store electric charge.

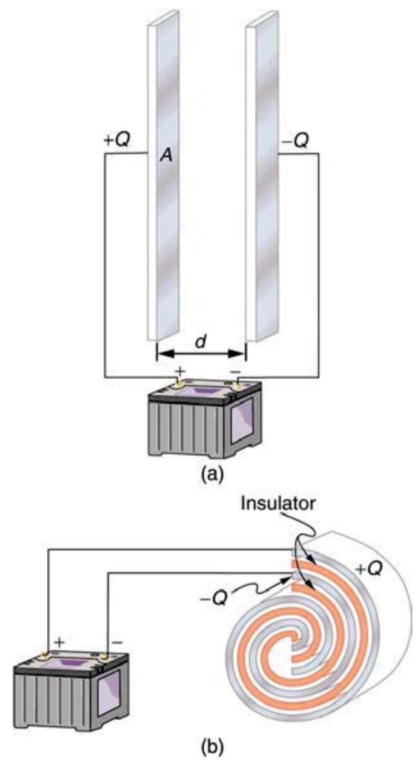

Figure 19.13 Both capacitors shown here were initially uncharged before being connected to a battery. They now have separated charges of $+Q$ and $-Q$ on their two halves. (a) A parallel plate capacitor. (b) A rolled capacitor with an insulating material between its two conducting sheets.

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.

The Amount of Charge Q a Capacitor Can Store

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.

A system composed of two identical, parallel conducting plates separated by a distance, as in **Figure 19.14**, is called a **parallel plate capacitor**. It is easy to see the relationship between the voltage and the stored charge for a parallel plate capacitor, as shown in **Figure 19.14**. Each electric field line starts on an individual positive charge and ends on a negative one, so that there will be more field lines if there is more charge. (Drawing a single field line per charge is a convenience, only. We can draw many field lines for each charge, but the total number is proportional to the number of charges.) The electric field strength is, thus, directly proportional to Q.

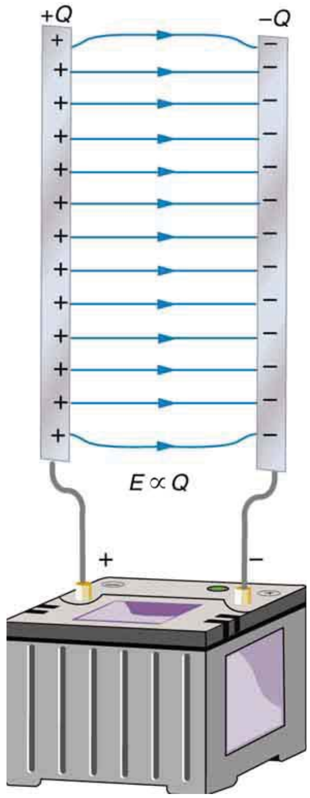

Figure 19.14 Electric field lines in this parallel plate capacitor, as always, start on positive charges and end on negative charges. Since the electric field strength is proportional to the density of field lines, it is also proportional to the amount of charge on the capacitor.

The field is proportional to the charge:

$$E \propto Q, \tag{19.45}$$

where the symbol $\propto$ means “proportional to.” From the discussion in **Electric Potential in a Uniform Electric Field**, we know that the voltage across parallel plates is $V = Ed$. Thus,

$$V \propto E. \tag{19.46}$$

It follows, then, that V Q , and conversely,

$$Q \propto V. \tag{19.47}$$

This is true in general: The greater the voltage applied to any capacitor, the greater the charge stored in it.

Different capacitors will store different amounts of charge for the same applied voltage, depending on their physical characteristics. We define their **capacitance** C to be such that the charge Q stored in a capacitor is proportional to C . The charge stored in a capacitor is given by

$$Q = CV. \tag{19.48}$$

This equation expresses the two major factors affecting the amount of charge stored. Those factors are the physical characteristics of the capacitor, C , and the voltage, V . Rearranging the equation, we see that *capacitance* C *is the amount of charge stored per volt,* or

$$C = \frac{Q}{V}. \tag{19.49}$$

Capacitance

Capacitance C is the amount of charge stored per volt, or

$$C = \frac{Q}{V}. \tag{19.50}$$

The unit of capacitance is the farad (F), named for Michael Faraday (1791–1867), an English scientist who contributed to the fields of electromagnetism and electrochemistry. Since capacitance is charge per unit voltage, we see that a farad is a coulomb per volt, or

$$1 \text{ F} = \frac{1 \text{ C}}{1 \text{ V}}. \tag{19.51}$$

A 1-farad capacitor would be able to store 1 coulomb (a very large amount of charge) with the application of only 1 volt. One farad is, thus, a very large capacitance. Typical capacitors range from fractions of a picofarad $\left(1 \text{ pF} = 10^{-12} \text{ F}\right)$ to millifarads $\left(1 \text{ mF} = 10^{-3} \text{ F}\right)$.

Figure 19.15 shows some common capacitors. Capacitors are primarily made of ceramic, glass, or plastic, depending upon purpose and size. Insulating materials, called dielectrics, are commonly used in their construction, as discussed below.

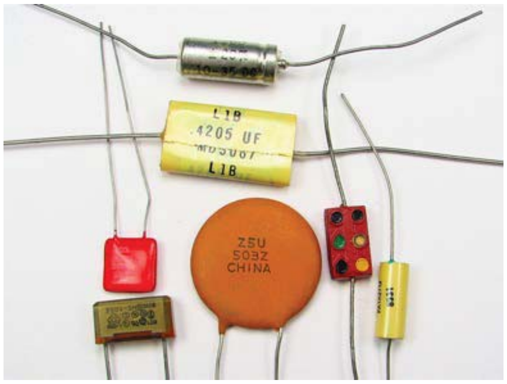

Figure 19.15 Some typical capacitors. Size and value of capacitance are not necessarily related. (credit: Windell Oskay)

Parallel Plate Capacitor

The parallel plate capacitor shown in **Figure 19.16** has two identical conducting plates, each having a surface area A , separated by a distance d (with no material between the plates). When a voltage V is applied to the capacitor, it stores a charge Q , as shown. We can see how its capacitance depends on A and d by considering the characteristics of the Coulomb force. We know that like charges repel, unlike charges attract, and the force between charges decreases with distance. So it seems quite reasonable that the bigger the plates are, the more charge they can store—because the charges can spread out more. Thus C should be greater for larger A . Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. So C should be greater for smaller d .

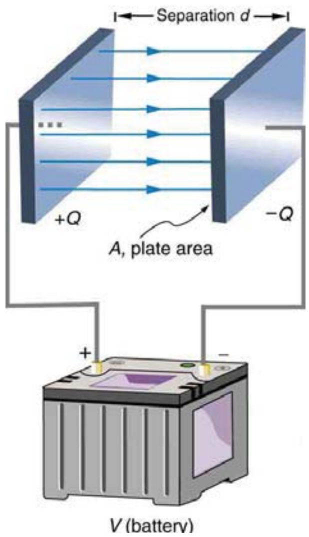

Figure 19.16 Parallel plate capacitor with plates separated by a distance d . Each plate has an area A .

It can be shown that for a parallel plate capacitor there are only two factors (A and d) that affect its capacitance C . The capacitance of a parallel plate capacitor in equation form is given by

$$C = \varepsilon_0 \frac{A}{d}. \tag{19.52}$$

Capacitance of a Parallel Plate Capacitor

$$C = \varepsilon_0 \frac{A}{d} \tag{19.53}$$

A is the area of one plate in square meters, and d is the distance between the plates in meters. The constant ε_0 is the permittivity of free space; its numerical value in SI units is $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m}$. The units of F/m are equivalent to $\text{C}^2/\text{N} \cdot \text{m}^2$. The small numerical value of ε_0 is related to the large size of the farad. A parallel plate capacitor must have a large area to have a capacitance approaching a farad. (Note that the above equation is valid when the parallel plates are separated by air or free space. When another material is placed between the plates, the equation is modified, as discussed below.)

Example 19.8 Capacitance and Charge Stored in a Parallel Plate Capacitor

(a) What is the capacitance of a parallel plate capacitor with metal plates, each of area $1.00\ \text{m}^2$, separated by 1.00 mm? (b) What charge is stored in this capacitor if a voltage of $3.00\times10^3\ \text{V}$ is applied to it?

Strategy

Finding the capacitance C is a straightforward application of the equation $C = \varepsilon_0 A / d$. Once C is found, the charge stored can be found using the equation $Q = CV$.

Solution for (a)

Entering the given values into the equation for the capacitance of a parallel plate capacitor yields

$$\begin{aligned} C &= \varepsilon_0 \frac{A}{d} = \left(8.85\times10^{-12}\ \frac{\text{F}}{\text{m}}\right)\frac{1.00\ \text{m}^2}{1.00\times10^{-3}\ \text{m}} \\ &= 8.85\times10^{-9}\ \text{F} = 8.85\ \text{nF}. \end{aligned} \tag{19.54}$$

Discussion for (a)

This small value for the capacitance indicates how difficult it is to make a device with a large capacitance. Special techniques help, such as using very large area thin foils placed close together.

Solution for (b)

The charge stored in any capacitor is given by the equation $Q = CV$. Entering the known values into this equation gives

$$\begin{aligned} Q &= CV = \left(8.85\times10^{-9}\ \text{F}\right)\left(3.00\times10^3\ \text{V}\right) \\ &= 26.6\ \mu\text{C}. \end{aligned} \tag{19.55}$$

Discussion for (b)

This charge is only slightly greater than those found in typical static electricity. Since air breaks down at about $3.00\times10^6\ \text{V/m}$, more charge cannot be stored on this capacitor by increasing the voltage.

Another interesting biological example dealing with electric potential is found in the cell's plasma membrane. The membrane sets a cell off from its surroundings and also allows ions to selectively pass in and out of the cell. There is a potential difference across the membrane of about $-70\ \text{mV}$. This is due to the mainly negatively charged ions in the cell and the predominance of positively charged sodium (Na^+) ions outside. Things change when a nerve cell is stimulated. Na^+ ions are allowed to pass through the membrane into the cell, producing a positive membrane potential—the nerve signal. The cell membrane is about 7 to 10 nm thick. An approximate value of the electric field across it is given by

$$E = \frac{V}{d} = \frac{-70\times10^{-3}\ \text{V}}{8\times10^{-9}\ \text{m}} = -9\times10^6\ \text{V/m}. \tag{19.56}$$

This electric field is enough to cause a breakdown in air.

Dielectric

The previous example highlights the difficulty of storing a large amount of charge in capacitors. If d is made smaller to produce a larger capacitance, then the maximum voltage must be reduced proportionally to avoid breakdown (since $E = V / d$). An important solution to this difficulty is to put an insulating material, called a **dielectric**, between the plates of a capacitor and allow d to be as small as possible. Not only does the smaller d make the capacitance greater, but many insulators can withstand greater electric fields than air before breaking down.

There is another benefit to using a dielectric in a capacitor. Depending on the material used, the capacitance is greater than that given by the equation $C = \varepsilon_0 \frac{A}{d}$ by a factor κ, called the *dielectric constant*. A parallel plate capacitor with a dielectric between its plates has a capacitance given by

$$C = \kappa\varepsilon_0 \frac{A}{d}\ \text{(parallel plate capacitor with dielectric)}. \tag{19.57}$$

Values of the dielectric constant κ for various materials are given in **Table 19.1**. Note that κ for vacuum is exactly 1, and so the above equation is valid in that case, too. If a dielectric is used, perhaps by placing Teflon between the plates of the capacitor in **Example 19.8**, then the capacitance is greater by the factor κ, which for Teflon is 2.1.

Take-Home Experiment: Building a Capacitor

How large a capacitor can you make using a chewing gum wrapper? The plates will be the aluminum foil, and the separation (dielectric) in between will be the paper.

Table 19.1 Dielectric Constants and Dielectric Strengths for Various Materials at 20ºC

Material	Dielectric constant κ	Dielectric strength (V/m)
Vacuum	1.00000	—
Air	1.00059	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Polystyrene	2.56	24×10^6
Pyrex glass	5.6	14×10^6
Silicon oil	2.5	15×10^6
Strontium titanate	233	8×10^6
Teflon	2.1	60×10^6
Water	80	—

Note also that the dielectric constant for air is very close to 1, so that air-filled capacitors act much like those with vacuum between their plates *except* that the air can become conductive if the electric field strength becomes too great. (Recall that $E = V/d$ for a parallel plate capacitor.) Also shown in **Table 19.1** are maximum electric field strengths in V/m, called **dielectric strengths**, for several materials. These are the fields above which the material begins to break down and conduct. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation. For instance, in **Example 19.8**, the separation is 1.00 mm, and so the voltage limit for air is

$$\begin{aligned} V &= E \cdot d \\ &= (3\times10^6 \text{ V/m})(1.00\times10^{-3} \text{ m}) \\ &= 3000 \text{ V}. \end{aligned} \tag{19.58}$$

However, the limit for a 1.00 mm separation filled with Teflon is 60,000 V, since the dielectric strength of Teflon is 60×10^6 V/m. So the same capacitor filled with Teflon has a greater capacitance and can be subjected to a much greater voltage. Using the capacitance we calculated in the above example for the air-filled parallel plate capacitor, we find that the Teflon-filled capacitor can store a maximum charge of

$$\begin{aligned} Q &= CV \\ &= \kappa C_{\text{air}} V \\ &= (2.1)(8.85 \text{ nF})(6.0\times10^4 \text{ V}) \\ &= 1.1 \text{ mC}. \end{aligned} \tag{19.59}$$

This is 42 times the charge of the same air-filled capacitor.

Dielectric Strength

The maximum electric field strength above which an insulating material begins to break down and conduct is called its dielectric strength.

Microscopically, how does a dielectric increase capacitance? Polarization of the insulator is responsible. The more easily it is polarized, the greater its dielectric constant κ. Water, for example, is a **polar molecule** because one end of the molecule has a slight positive charge and the other end has a slight negative charge. The polarity of water causes it to have a relatively large dielectric constant of 80. The effect of polarization can be best explained in terms of the characteristics of the Coulomb force. **Figure 19.17** shows the separation of charge schematically in the molecules of a dielectric material placed between the charged plates of a capacitor. The Coulomb force between the closest ends of the molecules and the charge on the plates is attractive and very strong, since they are very close together. This attracts more charge onto the plates than if the space were empty and the opposite charges were a distance d away.

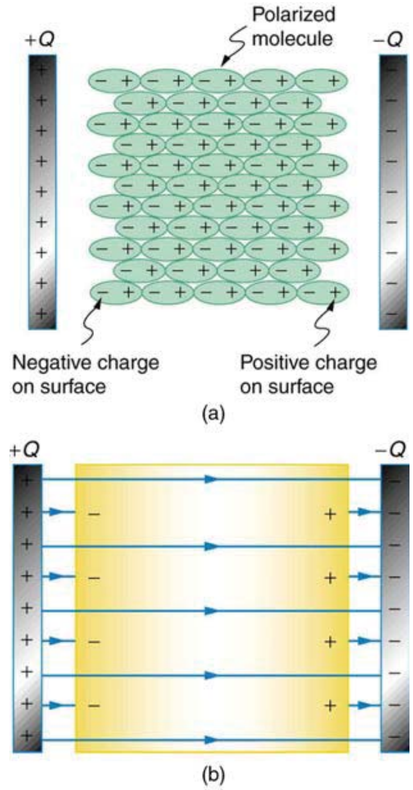

Figure 19.17 (a) The molecules in the insulating material between the plates of a capacitor are polarized by the charged plates. This produces a layer of opposite charge on the surface of the dielectric that attracts more charge onto the plate, increasing its capacitance. (b) The dielectric reduces the electric field strength inside the capacitor, resulting in a smaller voltage between the plates for the same charge. The capacitor stores the same charge for a smaller voltage, implying that it has a larger capacitance because of the dielectric.

Another way to understand how a dielectric increases capacitance is to consider its effect on the electric field inside the capacitor. **Figure 19.17**(b) shows the electric field lines with a dielectric in place. Since the field lines end on charges in the dielectric, there are fewer of them going from one side of the capacitor to the other. So the electric field strength is less than if there were a vacuum between the plates, even though the same charge is on the plates. The voltage between the plates is $V = Ed$, so it too is reduced by the dielectric. Thus there is a smaller voltage V for the same charge Q; since $C = Q/V$, the capacitance C is greater.

The dielectric constant is generally defined to be $\kappa = E_0/E$, or the ratio of the electric field in a vacuum to that in the dielectric material, and is intimately related to the polarizability of the material.

Things Great and Small

The Submicroscopic Origin of Polarization

Polarization is a separation of charge within an atom or molecule. As has been noted, the planetary model of the atom pictures it as having a positive nucleus orbited by negative electrons, analogous to the planets orbiting the Sun. Although this model is not completely accurate, it is very helpful in explaining a vast range of phenomena and will be refined elsewhere, such as in **Atomic Physics**. The submicroscopic origin of polarization can be modeled as shown in **Figure 19.18**.

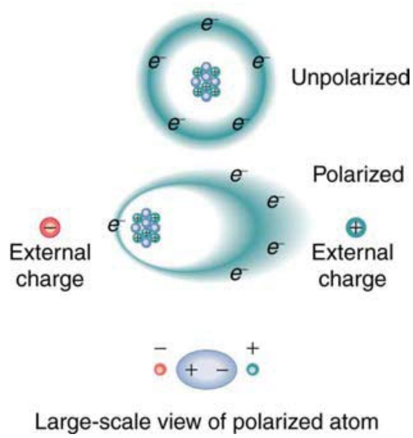

Figure 19.18 Artist's conception of a polarized atom. The orbits of electrons around the nucleus are shifted slightly by the external charges (shown exaggerated). The resulting separation of charge within the atom means that it is polarized. Note that the unlike charge is now closer to the external charges, causing the polarization.

We will find in **Atomic Physics** that the orbits of electrons are more properly viewed as electron clouds with the density of the cloud related to the probability of finding an electron in that location (as opposed to the definite locations and paths of planets in their orbits around the Sun). This cloud is shifted by the Coulomb force so that the atom on average has a separation of charge. Although the atom remains neutral, it can now be the source of a Coulomb force, since a charge brought near the atom will be closer to one type of charge than the other.

Some molecules, such as those of water, have an inherent separation of charge and are thus called polar molecules. **Figure 19.19** illustrates the separation of charge in a water molecule, which has two hydrogen atoms and one oxygen atom (H_2O). The water molecule is not symmetric—the hydrogen atoms are repelled to one side, giving the molecule a boomerang shape. The electrons in a water molecule are more concentrated around the more highly charged oxygen nucleus than around the hydrogen nuclei. This makes the oxygen end of the molecule slightly negative and leaves the hydrogen ends slightly positive. The inherent separation of charge in polar molecules makes it easier to align them with external fields and charges. Polar molecules therefore exhibit greater polarization effects and have greater dielectric constants. Those who study chemistry will find that the polar nature of water has many effects. For example, water molecules gather ions much more effectively because they have an electric field and a separation of charge to attract charges of both signs. Also, as brought out in the previous chapter, polar water provides a shield or screening of the electric fields in the highly charged molecules of interest in biological systems.

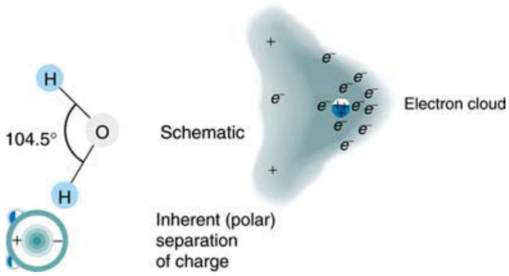

Figure 19.19 Artist's conception of a water molecule. There is an inherent separation of charge, and so water is a polar molecule. Electrons in the molecule are attracted to the oxygen nucleus and leave an excess of positive charge near the two hydrogen nuclei. (Note that the schematic on the right is a rough illustration of the distribution of electrons in the water molecule. It does not show the actual numbers of protons and electrons involved in the structure.)

PhET Explorations: Capacitor Lab

Explore how a capacitor works! Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electric field in the capacitor. Measure the voltage and the electric field.

PhET Interactive Simulation

Figure 19.20 Capacitor Lab (http://cnx.org/content/m42333/1.4/capacitor-lab_en.jar)

19.6 Capacitors in Series and Parallel

Several capacitors may be connected together in a variety of applications. Multiple connections of capacitors act like a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. There are two simple and common types of connections, called *series* and *parallel*, for which we can easily calculate the total capacitance. Certain more complicated connections can also be related to combinations of series and parallel.

Capacitance in Series

Figure 19.21(a) shows a series connection of three capacitors with a voltage applied. As for any capacitor, the capacitance of the combination is related to charge and voltage by $C = \frac{Q}{V}$.

Note in **Figure 19.21** that opposite charges of magnitude Q flow to either side of the originally uncharged combination of capacitors when the voltage V is applied. Conservation of charge requires that equal-magnitude charges be created on the plates of the individual capacitors, since charge is only being separated in these originally neutral devices. The end result is that the combination resembles a single capacitor with an effective plate separation greater than that of the individual capacitors alone. (See **Figure 19.21**(b).) Larger plate separation means smaller capacitance. It is a general feature of series connections of capacitors that the total capacitance is less than any of the individual capacitances.

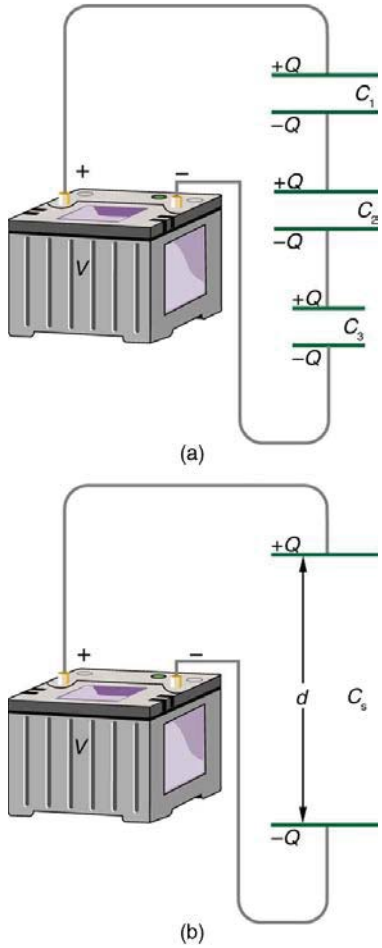

Figure 19.21 (a) Capacitors connected in series. The magnitude of the charge on each plate is Q. (b) An equivalent capacitor has a larger plate separation d. Series connections produce a total capacitance that is less than that of any of the individual capacitors.

We can find an expression for the total capacitance by considering the voltage across the individual capacitors shown in **Figure 19.21**. Solving $C = \frac{Q}{V}$ for V gives $V = \frac{Q}{C}$. The voltages across the individual capacitors are thus $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, and $V_3 = \frac{Q}{C_3}$. The total voltage is the sum of the individual voltages:

$$V = V_1 + V_2 + V_3. \tag{19.60}$$

Now, calling the total capacitance C_S for series capacitance, consider that

$$V = \frac{Q}{C_\text{S}} = V_1 + V_2 + V_3\,. \tag{19.61}$$

Entering the expressions for V_1, V_2, and V_3, we get

$$\frac{Q}{C_\text{S}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}. \tag{19.62}$$

Canceling the Q s, we obtain the equation for the total capacitance in series C_S to be

$$\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ..., \tag{19.63}$$

where "..." indicates that the expression is valid for any number of capacitors connected in series. An expression of this form always results in a total capacitance C_S that is less than any of the individual capacitances C_1, C_2, ..., as the next example illustrates.

Total Capacitance in Series, C_s

Total capacitance in series: $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$

Example 19.9 What Is the Series Capacitance?

Find the total capacitance for three capacitors connected in series, given their individual capacitances are 1.000, 5.000, and 8.000 μF.

Strategy

With the given information, the total capacitance can be found using the equation for capacitance in series.

Solution

Entering the given capacitances into the expression for $\frac{1}{C_S}$ gives $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$.

$$\frac{1}{C_S} = \frac{1}{1.000\ \mu F} + \frac{1}{5.000\ \mu F} + \frac{1}{8.000\ \mu F} = \frac{1.325}{\mu F} \tag{19.64}$$

Inverting to find C_S yields $C_S = \frac{\mu F}{1.325} = 0.755\ \mu F$.

Discussion

The total series capacitance C_s is less than the smallest individual capacitance, as promised. In series connections of capacitors, the sum is less than the parts. In fact, it is less than any individual. Note that it is sometimes possible, and more convenient, to solve an equation like the above by finding the least common denominator, which in this case (showing only whole-number calculations) is 40. Thus,

$$\frac{1}{C_S} = \frac{40}{40\ \mu F} + \frac{8}{40\ \mu F} + \frac{5}{40\ \mu F} = \frac{53}{40\ \mu F}, \tag{19.65}$$

so that

$$C_S = \frac{40\ \mu F}{53} = 0.755\ \mu F. \tag{19.66}$$

Capacitors in Parallel

Figure 19.22(a) shows a parallel connection of three capacitors with a voltage applied. Here the total capacitance is easier to find than in the series case. To find the equivalent total capacitance C_p, we first note that the voltage across each capacitor is V, the same as that of the source, since they are connected directly to it through a conductor. (Conductors are equipotentials, and so the voltage across the capacitors is the same as that across the voltage source.) Thus the capacitors have the same charges on them as they would have if connected individually to the voltage source. The total charge Q is the sum of the individual charges:

$$Q = Q_1 + Q_2 + Q_3. \tag{19.67}$$

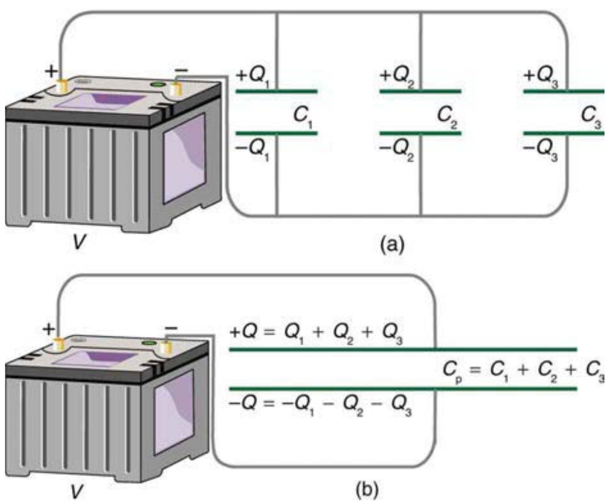

Figure 19.22 (a) Capacitors in parallel. Each is connected directly to the voltage source just as if it were all alone, and so the total capacitance in parallel is just the sum of the individual capacitances. (b) The equivalent capacitor has a larger plate area and can therefore hold more charge than the individual capacitors.

Using the relationship $Q = CV$, we see that the total charge is $Q = C_p V$, and the individual charges are $Q_1 = C_1 V$, $Q_2 = C_2 V$, and $Q_3 = C_3 V$. Entering these into the previous equation gives

$$C_p V = C_1 V + C_2 V + C_3 V. \tag{19.68}$$

Canceling V from the equation, we obtain the equation for the total capacitance in parallel C_p:

$$C_p = C_1 + C_2 + C_3 + ... \tag{19.69}$$

Total capacitance in parallel is simply the sum of the individual capacitances. (Again the "..." indicates the expression is valid for any number of capacitors connected in parallel.) So, for example, if the capacitors in the example above were connected in parallel, their capacitance would be

$$C_\text{p} = 1.000\ \mu\text{F}+5.000\ \mu\text{F}+8.000\ \mu\text{F} = 14.000\ \mu\text{F}. \tag{19.70}$$

The equivalent capacitor for a parallel connection has an effectively larger plate area and, thus, a larger capacitance, as illustrated in **Figure 19.22**(b).

Total Capacitance in Parallel, C_p

Total capacitance in parallel $C_\text{p} = C_1 + C_2 + C_3 + ...$

More complicated connections of capacitors can sometimes be combinations of series and parallel. (See **Figure 19.23**.) To find the total capacitance of such combinations, we identify series and parallel parts, compute their capacitances, and then find the total.

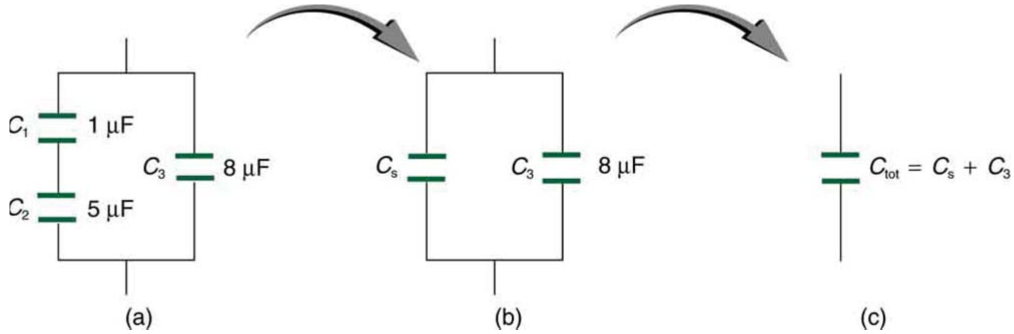

Figure 19.23 (a) This circuit contains both series and parallel connections of capacitors. See **Example 19.10** for the calculation of the overall capacitance of the circuit. (b) C_1 and C_2 are in series; their equivalent capacitance C_S is less than either of them. (c) Note that C_S is in parallel with C_3. The total capacitance is, thus, the sum of C_S and C_3.

Example 19.10 A Mixture of Series and Parallel Capacitance

Find the total capacitance of the combination of capacitors shown in **Figure 19.23**. Assume the capacitances in **Figure 19.23** are known to three decimal places ($C_1 = 1.000\ \mu\text{F}$, $C_2 = 3.000\ \mu\text{F}$, and $C_3 = 8.000\ \mu\text{F}$), and round your answer to three decimal places.

Strategy

To find the total capacitance, we first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S in the figure, is in parallel with C_3.

Solution

Since C_1 and C_2 are in series, their total capacitance is given by $\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. Entering their values into the equation gives

$$\frac{1}{C_\text{S}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000\ \mu\text{F}} + \frac{1}{5.000\ \mu\text{F}} = \frac{1.200}{\mu\text{F}}. \tag{19.71}$$

Inverting gives

$$C_\text{S} = 0.833\ \mu\text{F}. \tag{19.72}$$

This equivalent series capacitance is in parallel with the third capacitor; thus, the total is the sum

$$\begin{aligned} C_\text{tot} &= C_\text{S} + C_\text{S} \\ &= 0.833\ \mu\text{F} + 8.000\ \mu\text{F} \\ &= 8.833\ \mu\text{F}. \end{aligned} \tag{19.73}$$

Discussion

This technique of analyzing the combinations of capacitors piece by piece until a total is obtained can be applied to larger combinations of capacitors.

19.7 Energy Stored in Capacitors

Most of us have seen dramatizations in which medical personnel use a **defibrillator** to pass an electric current through a patient's heart to get it to beat normally. (Review **Figure 19.24**.) Often realistic in detail, the person applying the shock directs another person to "make it 400 joules this time." The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics, such as certain handheld calculators, to supply energy when batteries are charged. (See **Figure 19.24**.) Capacitors are also used to supply energy for flash lamps on cameras.

Figure 19.24 Energy stored in the large capacitor is used to preserve the memory of an electronic calculator when its batteries are charged. (credit: Kucharek, Wikimedia Commons)

Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge Q and voltage V on the capacitor. We must be careful when applying the equation for electrical potential energy $\Delta\text{PE} = q\Delta V$ to a capacitor. Remember that ΔPE is the potential energy of a charge q going through a voltage ΔV. But the capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage $\Delta V = 0$, since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences $\Delta V = V$, since the capacitor now has its full voltage V on it. The average voltage on the capacitor during the charging process is $V/2$, and so the average voltage experienced by the full charge q is $V/2$. Thus the energy stored in a capacitor, E_{cap}, is

$$E_{\text{cap}} = \frac{QV}{2}, \tag{19.74}$$

where Q is the charge on a capacitor with a voltage V applied. (Note that the energy is not QV, but $QV/2$.) Charge and voltage are related to the capacitance C of a capacitor by $Q = CV$, and so the expression for E_{cap} can be algebraically manipulated into three equivalent expressions:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}, \tag{19.75}$$

where Q is the charge and V the voltage on a capacitor C. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

Energy Stored in Capacitors

The energy stored in a capacitor can be expressed in three ways:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C}, \tag{19.76}$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

In a defibrillator, the delivery of a large charge in a short burst to a set of paddles across a person's chest can be a lifesaver. The person's heart attack might have arisen from the onset of fast, irregular beating of the heart—cardiac or ventricular fibrillation. The application of a large shock of electrical energy can terminate the arrhythmia and allow the body's pacemaker to resume normal patterns. Today it is common for ambulances to carry a defibrillator, which also uses an electrocardiogram to analyze the patient's heartbeat pattern. Automated external defibrillators (AED) are found in many public places (**Figure 19.25**). These are designed to be used by lay persons. The device automatically diagnoses the patient's heart condition and then applies the shock with appropriate energy and waveform. CPR is recommended in many cases before use of an AED.

Figure 19.25 Automated external defibrillators are found in many public places. These portable units provide verbal instructions for use in the important first few minutes for a person suffering a cardiac attack. (credit: Owain Davies, Wikimedia Commons)

Example 19.11 Capacitance in a Heart Defibrillator

A heart defibrillator delivers $4.00\times10^{2}\text{ J}$ of energy by discharging a capacitor initially at $1.00\times10^{4}\text{ V}$. What is its capacitance?

Strategy

We are given E_{cap} and V, and we are asked to find the capacitance C. Of the three expressions in the equation for E_{cap}, the most convenient relationship is

$$E_{\text{cap}} = \frac{CV^2}{2}. \quad (19.77)$$

Solution

Solving this expression for C and entering the given values yields

$$\begin{aligned} C &= \frac{2E_{\text{cap}}}{V^2} = \frac{2(4.00\times10^{2}\text{ J})}{(1.00\times10^{4}\text{ V})^2} = 8.00\times10^{-6}\text{ F} \\ &= 8.00\ \mu\text{F}. \end{aligned} \quad (19.78)$$

Discussion

This is a fairly large, but manageable, capacitance at $1.00\times10^{4}\text{ V}$.

Glossary

capacitance: amount of charge stored per unit volt

capacitor: a device that stores electric charge

defibrillator: a machine used to provide an electrical shock to a heart attack victim's heart in order to restore the heart's normal rhythmic pattern

dielectric strength: the maximum electric field above which an insulating material begins to break down and conduct

dielectric: an insulating material

electric potential: potential energy per unit charge

electron volt: the energy given to a fundamental charge accelerated through a potential difference of one volt

equipotential line: a line along which the electric potential is constant

grounding: fixing a conductor at zero volts by connecting it to the earth or ground

mechanical energy: sum of the kinetic energy and potential energy of a system; this sum is a constant

parallel plate capacitor: two identical conducting plates separated by a distance

polar molecule: a molecule with inherent separation of charge

potential difference (or voltage): change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

scalar: physical quantity with magnitude but no direction

vector: physical quantity with both magnitude and direction

Section Summary

19.1 Electric Potential Energy: Potential Difference

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol ΔV.

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

$$\begin{aligned} 1\text{ eV} &= \left(1.60\times10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60\times10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60\times10^{-19}\text{ J.} \end{aligned}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $\text{KE} + \text{PE}.$ This sum is a constant.

19.2 Electric Potential in a Uniform Electric Field

- The voltage between points A and B is

$$\left.\begin{aligned} V_{\text{AB}} &= Ed \\ E &= \frac{V_{\text{AB}}}{d} \end{aligned}\right\}(\text{uniform } E\text{ - field on y}),$$

  where d is the distance from A to B, or the distance between the plates.
- In equation form, the general relationship between voltage and electric field is

$$E = -\frac{\Delta V}{\Delta s},$$

  where Δs is the distance over which the change in potential, ΔV, takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

19.3 Electrical Potential Due to a Point Charge

- Electric potential of a point charge is $V = kQ/r$.
- Electric potential is a scalar, and electric field is a vector. Addition of voltages as numbers gives the voltage due to a combination of point charges, whereas addition of individual fields as vectors gives the total electric field.

19.4 Equipotential Lines

- An equipotential line is a line along which the electric potential is constant.
- An equipotential surface is a three-dimensional version of equipotential lines.
- Equipotential lines are always perpendicular to electric field lines.
- The process by which a conductor can be fixed at zero volts by connecting it to the earth with a good conductor is called grounding.

19.5 Capacitors and Dielectrics

- A capacitor is a device used to store charge.
- The amount of charge Q a capacitor can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.
- The capacitance C is the amount of charge stored per volt, or

$$C = \frac{Q}{V}.$$

- The capacitance of a parallel plate capacitor is $C = \varepsilon_0 \frac{A}{d}$, when the plates are separated by air or free space. ε_0 is called the permittivity of free space.
- A parallel plate capacitor with a dielectric between its plates has a capacitance given by

$$C = \kappa\varepsilon_0 \frac{A}{d},$$

  where κ is the dielectric constant of the material.
- The maximum electric field strength above which an insulating material begins to break down and conduct is called dielectric strength.

19.6 Capacitors in Series and Parallel

- Total capacitance in series $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$
- Total capacitance in parallel $C_p = C_1 + C_2 + C_3 + ...$
- If a circuit contains a combination of capacitors in series and parallel, identify series and parallel parts, compute their capacitances, and then find the total.

19.7 Energy Stored in Capacitors

- Capacitors are used in a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps, to supply energy.
- The energy stored in a capacitor can be expressed in three ways:

$$E_{cap} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules when the charge is in coulombs, voltage is in volts, and capacitance is in farads.

Conceptual Questions

19.1 Electric Potential Energy: Potential Difference

1. Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

2. If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

3. What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

4. Voltages are always measured between two points. Why?

5. How are units of volts and electron volts related? How do they differ?

19.2 Electric Potential in a Uniform Electric Field

6. Discuss how potential difference and electric field strength are related. Give an example.

7. What is the strength of the electric field in a region where the electric potential is constant?

8. Will a negative charge, initially at rest, move toward higher or lower potential? Explain why.

19.3 Electrical Potential Due to a Point Charge

9. In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

10. Can the potential of a non-uniformly charged sphere be the same as that of a point charge? Explain.

19.4 Equipotential Lines

11. What is an equipotential line? What is an equipotential surface?

12. Explain in your own words why equipotential lines and surfaces must be perpendicular to electric field lines.

13. Can different equipotential lines cross? Explain.

19.5 Capacitors and Dielectrics

14. Does the capacitance of a device depend on the applied voltage? What about the charge stored in it?

15. Use the characteristics of the Coulomb force to explain why capacitance should be proportional to the plate area of a capacitor. Similarly, explain why capacitance should be inversely proportional to the separation between plates.

16. Give the reason why a dielectric material increases capacitance compared with what it would be with air between the plates of a capacitor. What is the independent reason that a dielectric material also allows a greater voltage to be applied to a capacitor? (The dielectric thus increases C and permits a greater V.)

17. How does the polar character of water molecules help to explain water's relatively large dielectric constant? (**Figure 19.19**)

18. Sparks will occur between the plates of an air-filled capacitor at lower voltage when the air is humid than when dry. Explain why, considering the polar character of water molecules.

19. Water has a large dielectric constant, but it is rarely used in capacitors. Explain why.

20. Membranes in living cells, including those in humans, are characterized by a separation of charge across the membrane. Effectively, the membranes are thus charged capacitors with important functions related to the potential difference across the membrane. Is energy required to separate these charges in living membranes and, if so, is its source the metabolization of food energy or some other source?

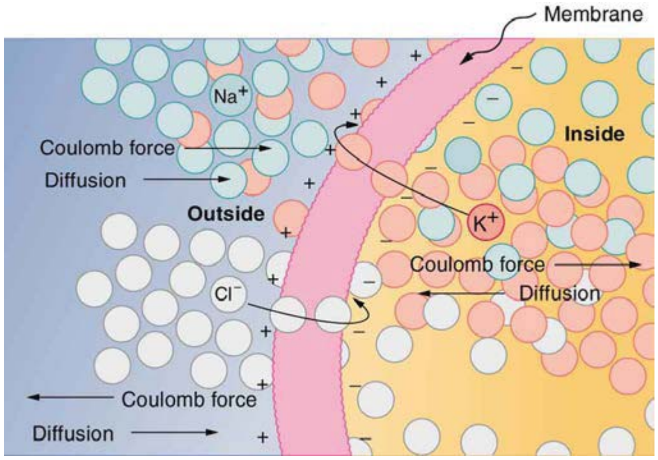

Figure 19.26 The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na^+ (sodium ions).

19.6 Capacitors in Series and Parallel

21. If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

19.7 Energy Stored in Capacitors

22. How does the energy contained in a charged capacitor change when a dielectric is inserted, assuming the capacitor is isolated and its charge is constant? Does this imply that work was done?

23. What happens to the energy stored in a capacitor connected to a battery when a dielectric is inserted? Was work done in the process?

Problems & Exercises

19.1 Electric Potential Energy: Potential Difference

1. Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be $1.67\times10^{-27}\text{ kg}$.

2. An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

3. A bare helium nucleus has two positive charges and a mass of $6.64\times10^{-27}\text{ kg}$. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

4. Integrated Concepts

Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

5. Integrated Concepts

The temperature near the center of the Sun is thought to be 15 million degrees Celsius $\left(1.5\times10^{7}\ ^{\circ}\text{C}\right)$. Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

6. Integrated Concepts

(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

7. Integrated Concepts

A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of $1.00\times10^{2}\text{ MV}$. (a) What energy was dissipated? (b) What mass of water could be raised from 15°C to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

8. Integrated Concepts

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, $2.50\times10^{2}\text{ g}$ of baby formula, and $2.00\times10^{2}\text{ g}$ of aluminum from 20.0°C to 90.0°C. (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

9. Integrated Concepts

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a $2.00\times10^{2}\text{ m}$ high hill, and then cause it to travel at a constant 25.0 m/s by exerting a $5.00\times10^{2}\text{ N}$ force for an hour.

10. Integrated Concepts

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another. (a) Calculate the potential energy of two singly charged nuclei separated by $1.00\times10^{-12}\text{ m}$ by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

11. Unreasonable Results

(a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

12. Construct Your Own Problem

Consider a battery used to supply energy to a cellular phone. Construct a problem in which you determine the energy that must be supplied by the battery, and then calculate the amount of charge it must be able to move in order to supply this energy. Among the things to be considered are the energy needs and battery voltage. You may need to look ahead to interpret manufacturer's battery ratings in ampere-hours as energy in joules.

19.2 Electric Potential in a Uniform Electric Field

13. Show that units of V/m and N/C for electric field strength are indeed equivalent.

14. What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of $1.50\times10^{4}\text{ V}$?

15. The electric field strength between two parallel conducting plates separated by 4.00 cm is $7.50\times10^{4}\text{ V/m}$. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)?

16. How far apart are two conducting plates that have an electric field strength of $4.50\times10^{3}\text{ V/m}$ between them, if their potential difference is 15.0 kV?

17. (a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air ($3.0\times10^{6}\text{ V/m}$) if the plates are separated by 2.00 mm and a potential difference of $5.0\times10^{3}\text{ V}$ is applied? (b) How close together can the plates be with this applied voltage?

18. The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct. Membranes are discussed in **Capacitors and Dielectrics** and **Nerve Conduction—Electrocardiograms**.) You may assume a uniform electric field.

19. Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. (Membranes are discussed in some detail in **Nerve Conduction—Electrocardiograms**.) What is the voltage across an 8.00 nm–thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

20. Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

21. Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be $3.0\times10^{6}\text{ V/m}$.

22. A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

23. An electron is to be accelerated in a uniform electric field having a strength of $2.00\times10^{6}\text{ V/m}$. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

19.3 Electrical Potential Due to a Point Charge

24. A 0.500 cm diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0 pC charge on its surface. What is the potential near its surface?

25. What is the potential $0.530\times10^{-10}\text{ m}$ from a proton (the average distance between the proton and electron in a hydrogen atom)?

26. (a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

27. How far from a $1.00\ \mu\text{C}$ point charge will the potential be 100 V? At what distance will it be $2.00\times10^{2}\text{ V}$?

28. What are the sign and magnitude of a point charge that produces a potential of -2.00 V at a distance of 1.00 mm?

29. If the potential due to a point charge is $5.00\times10^{2}\text{ V}$ at a distance of 15.0 m, what are the sign and magnitude of the charge?

30. In nuclear fission, a nucleus splits roughly in half. (a) What is the potential $2.00\times10^{-14}\text{ m}$ from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?

31. A research Van de Graaff generator has a 2.00-m-diameter metal sphere with a charge of 5.00 mC on it. (a) What is the potential near its surface? (b) At what distance from its center is the potential 1.00 MV? (c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its energy in MeV at this distance?

32. An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object. (a) What charge is on the sphere? (b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?

33. In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

34. (a) What is the potential between two points situated 10 cm and 20 cm from a $3.0\ \mu\text{C}$ point charge? (b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

35. Unreasonable Results

(a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graaff terminal?

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

19.4 Equipotential Lines

36. (a) Sketch the equipotential lines near a point charge $+q$. Indicate the direction of increasing potential. (b) Do the same for a point charge $-3q$.

37. Sketch the equipotential lines for the two equal positive charges shown in **Figure 19.27**. Indicate the direction of increasing potential.

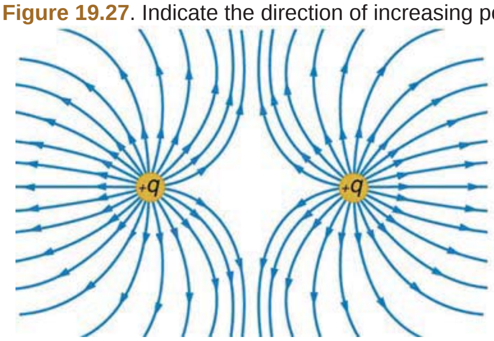

Figure 19.27 The electric field near two equal positive charges is directed away from each of the charges.

38. Figure 19.28 shows the electric field lines near two charges q_1 and q_2, the first having a magnitude four times that of the second. Sketch the equipotential lines for these two charges, and indicate the direction of increasing potential.

39. Sketch the equipotential lines a long distance from the charges shown in **Figure 19.28**. Indicate the direction of increasing potential.

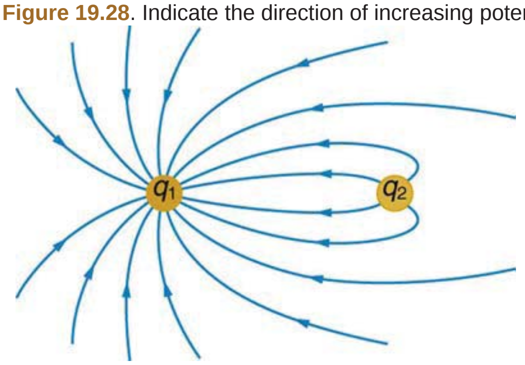

Figure 19.28 The electric field near two charges.

40. Sketch the equipotential lines in the vicinity of two opposite charges, where the negative charge is three times as great in magnitude as the positive. See **Figure 19.28** for a similar situation. Indicate the direction of increasing potential.

41. Sketch the equipotential lines in the vicinity of the negatively charged conductor in **Figure 19.29**. How will these equipotentials look a long distance from the object?

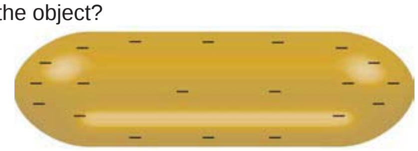

Figure 19.29 A negatively charged conductor.

42. Sketch the equipotential lines surrounding the two conducting plates shown in **Figure 19.30**, given the top plate is positive and the bottom plate has an equal amount of negative charge. Be certain to indicate the distribution of charge on the plates. Is the field strongest where the plates are closest? Why should it be?

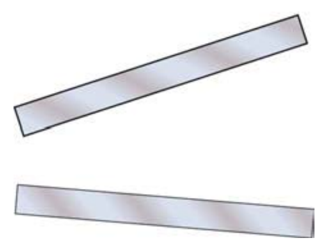

Figure 19.30

43. (a) Sketch the electric field lines in the vicinity of the charged insulator in **Figure 19.31**. Note its non-uniform charge distribution. (b) Sketch equipotential lines surrounding the insulator. Indicate the direction of increasing potential.

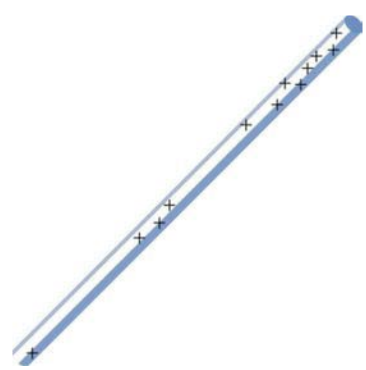

Figure 19.31 A charged insulating rod such as might be used in a classroom demonstration.

44. The naturally occurring charge on the ground on a fine day out in the open country is $-1.00\ \text{nC/m}^2$. (a) What is the electric field relative to ground at a height of 3.00 m? (b) Calculate the electric potential at this height. (c) Sketch electric field and equipotential lines for this scenario.

45. The lesser electric ray (*Narcine bancroftii*) maintains an incredible charge on its head and a charge equal in magnitude but opposite in sign on its tail (**Figure 19.32**). (a) Sketch the equipotential lines surrounding the ray. (b) Sketch the equipotentials when the ray is near a ship with a conducting surface. (c) How could this charge distribution be of use to the ray?

Figure 19.32 Lesser electric ray (*Narcine bancroftii*) (credit: National Oceanic and Atmospheric Administration, NOAA's Fisheries Collection).

19.5 Capacitors and Dielectrics

46. What charge is stored in a $180\ \mu\text{F}$ capacitor when 120 V is applied to it?

47. Find the charge stored when 5.50 V is applied to an 8.00 pF capacitor.

48. What charge is stored in the capacitor in **Example 19.8**?

49. Calculate the voltage applied to a $2.00\ \mu\text{F}$ capacitor when it holds $3.10\ \mu\text{C}$ of charge.

50. What voltage must be applied to an 8.00 nF capacitor to store 0.160 mC of charge?

51. What capacitance is needed to store $3.00\ \mu\text{C}$ of charge at a voltage of 120 V?

52. What is the capacitance of a large Van de Graaff generator's terminal, given that it stores 8.00 mC of charge at a voltage of 12.0 MV?

53. Find the capacitance of a parallel plate capacitor having plates of area $5.00\ \text{m}^2$ that are separated by 0.100 mm of Teflon.

54. (a)What is the capacitance of a parallel plate capacitor having plates of area $1.50\ \text{m}^2$ that are separated by 0.0200 mm of neoprene rubber? (b) What charge does it hold when 9.00 V is applied to it?

55. Integrated Concepts

A prankster applies 450 V to an $80.0\ \mu\text{F}$ capacitor and then tosses it to an unsuspecting victim. The victim's finger is burned by the discharge of the capacitor through 0.200 g of flesh. What is the temperature increase of the flesh? Is it reasonable to assume no phase change?

56. Unreasonable Results

(a) A certain parallel plate capacitor has plates of area $4.00\ \text{m}^2$, separated by 0.0100 mm of nylon, and stores 0.170 C of charge. What is the applied voltage? (b) What is unreasonable about this result? (c) Which assumptions are responsible or inconsistent?

19.6 Capacitors in Series and Parallel

57. Find the total capacitance of the combination of capacitors in **Figure 19.33**.

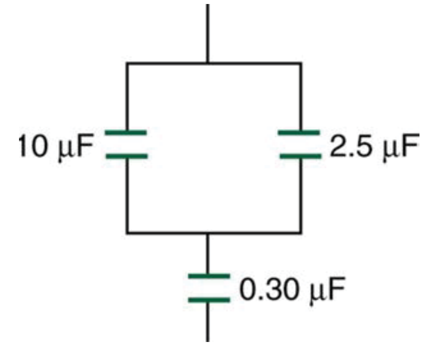

Figure 19.33 A combination of series and parallel connections of capacitors.

58. Suppose you want a capacitor bank with a total capacitance of 0.750 F and you possess numerous 1.50 mF capacitors. What is the smallest number you could hook together to achieve your goal, and how would you connect them?

59. What total capacitances can you make by connecting a $5.00\ \mu\text{F}$ and an $8.00\ \mu\text{F}$ capacitor together?

60. Find the total capacitance of the combination of capacitors shown in **Figure 19.34**.

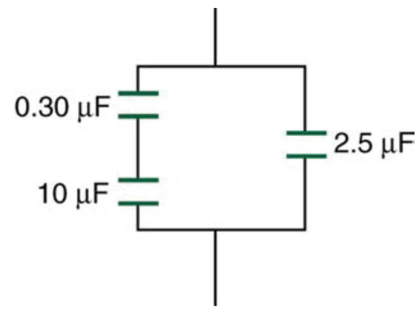

Figure 19.34 A combination of series and parallel connections of capacitors.

61. Find the total capacitance of the combination of capacitors shown in **Figure 19.35**.

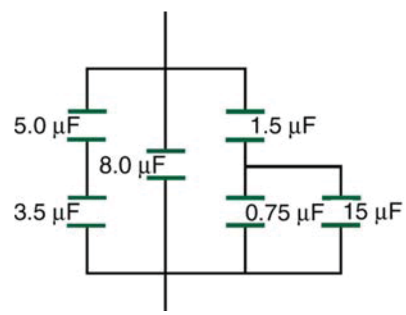

Figure 19.35 A combination of series and parallel connections of capacitors.

62. Unreasonable Results

(a) An $8.00\ \mu\text{F}$ capacitor is connected in parallel to another capacitor, producing a total capacitance of $5.00\ \mu\text{F}$. What is the capacitance of the second capacitor? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

19.7 Energy Stored in Capacitors

63. (a) What is the energy stored in the $10.0\ \mu\text{F}$ capacitor of a heart defibrillator charged to $9.00\times10^3\ \text{V}$? (b) Find the amount of stored charge.

64. In open heart surgery, a much smaller amount of energy will defibrillate the heart. (a) What voltage is applied to the $8.00\ \mu\text{F}$ capacitor of a heart defibrillator that stores 40.0 J of energy? (b) Find the amount of stored charge.

65. A $165\ \mu F$ capacitor is used in conjunction with a motor. How much energy is stored in it when 119 V is applied?

66. Suppose you have a 9.00 V battery, a $2.00\ \mu F$ capacitor, and a $7.40\ \mu F$ capacitor. (a) Find the charge and energy stored if the capacitors are connected to the battery in series. (b) Do the same for a parallel connection.

67. A nervous physicist worries that the two metal shelves of his wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction. (a) What is the capacitance of the empty shelves if they have area $1.00 \times 10^2\ m^2$ and are 0.200 m apart? (b) What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them? (c) To show that this voltage poses a small hazard, calculate the energy stored.

68. Show that for a given dielectric material the maximum energy a parallel plate capacitor can store is directly proportional to the volume of dielectric ($\text{Volume} = A \cdot d$). Note that the applied voltage is limited by the dielectric strength.

69. Construct Your Own Problem

Consider a heart defibrillator similar to that discussed in **Example 19.11**. Construct a problem in which you examine the charge stored in the capacitor of a defibrillator as a function of stored energy. Among the things to be considered are the applied voltage and whether it should vary with energy to be delivered, the range of energies involved, and the capacitance of the defibrillator. You may also wish to consider the much smaller energy needed for defibrillation during open-heart surgery as a variation on this problem.

70. Unreasonable Results

(a) On a particular day, it takes $9.60 \times 10^3\ J$ of electric energy to start a truck's engine. Calculate the capacitance of a capacitor that could store that amount of energy at 12.0 V. (b) What is unreasonable about this result? (c) Which assumptions are responsible?