6 | APPLICATIONS OF NEWTON'S LAWS

Figure 6.1 Stock cars racing in the Grand National Divisional race at Iowa Speedway in May, 2015. Cars often reach speeds of 200 mph (320 km/h).

Chapter Outline

Introduction

Car racing has grown in popularity in recent years. As each car moves in a curved path around the turn, its wheels also spin rapidly. The wheels complete many revolutions while the car makes only part of one (a circular arc). How can we describe the velocities, accelerations, and forces involved? What force keeps a racecar from spinning out, hitting the wall bordering the track? What provides this force? Why is the track banked? We answer all of these questions in this chapter as we expand our consideration of Newton’s laws of motion.

6.1 | Solving Problems with Newton's Laws

Learning Objectives

By the end of the section, you will be able to:

- Apply problem-solving techniques to solve for quantities in more complex systems of forces
- Use concepts from kinematics to solve problems using Newton's laws of motion
- Solve more complex equilibrium problems
- Solve more complex acceleration problems
- Apply calculus to more advanced dynamics problems

Success in problem solving is necessary to understand and apply physical principles. We developed a pattern of analyzing and setting up the solutions to problems involving Newton's laws in **Newton's Laws of Motion**; in this chapter, we continue to discuss these strategies and apply a step-by-step process.

Problem-Solving Strategies

We follow here the basics of problem solving presented earlier in this text, but we emphasize specific strategies that are useful in applying Newton's laws of motion. Once you identify the physical principles involved in the problem and determine that they include Newton's laws of motion, you can apply these steps to find a solution. These techniques also reinforce concepts that are useful in many other areas of physics. Many problem-solving strategies are stated outright in the worked examples, so the following techniques should reinforce skills you have already begun to develop.

Problem-Solving Strategy: Applying Newton's Laws of Motion

1. Identify the physical principles involved by listing the givens and the quantities to be calculated.
2. Sketch the situation, using arrows to represent all forces.
3. Determine the system of interest. The result is a free-body diagram that is essential to solving the problem.
4. Apply Newton's second law to solve the problem. If necessary, apply appropriate kinematic equations from the chapter on motion along a straight line.
5. Check the solution to see whether it is reasonable.

Let's apply this problem-solving strategy to the challenge of lifting a grand piano into a second-story apartment. Once we have determined that Newton's laws of motion are involved (if the problem involves forces), it is particularly important to draw a careful sketch of the situation. Such a sketch is shown in **Figure 6.2**(a). Then, as in **Figure 6.2**(b), we can represent all forces with arrows. Whenever sufficient information exists, it is best to label these arrows carefully and make the length and direction of each correspond to the represented force.

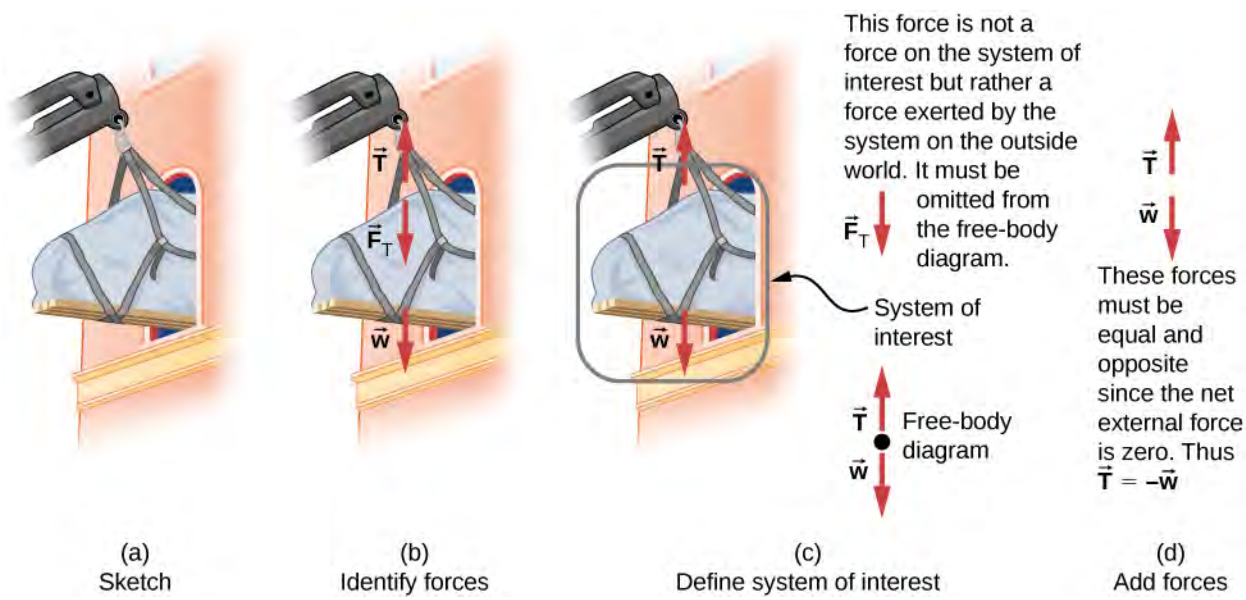


Figure 6.2 (a) A grand piano is being lifted to a second-story apartment. (b) Arrows are used to represent all forces: $\vec{\mathbf{T}}$ is the tension in the rope above the piano, $\vec{\mathbf{F}}_{\text{T}}$ is the force that the piano exerts on the rope, and $\vec{\mathbf{w}}$ is the weight of the piano. All other forces, such as the nudge of a breeze, are assumed to be negligible. (c) Suppose we are given the piano's mass and asked to find the tension in the rope. We then define the system of interest as shown and draw a free-body diagram. Now $\vec{\mathbf{F}}_{\text{T}}$ is no longer shown, because it is not a force acting on the system of interest; rather, $\vec{\mathbf{F}}_{\text{T}}$ acts on the outside world. (d) Showing only the arrows, the head-to-tail method of addition is used. It is apparent that if the piano is stationary, $\vec{\mathbf{T}} = -\vec{\mathbf{w}}$.

As with most problems, we next need to identify what needs to be determined and what is known or can be inferred from the problem as stated, that is, make a list of knowns and unknowns. It is particularly crucial to identify the system of interest, since Newton's second law involves only external forces. We can then determine which forces are external and which are internal, a necessary step to employ Newton's second law. (See **Figure 6.2**(c).) Newton's third law may be used to identify whether forces are exerted between components of a system (internal) or between the system and something outside (external). As illustrated in **Newton's Laws of Motion**, the system of interest depends on the question we need to answer. Only forces are shown in free-body diagrams, not acceleration or velocity. We have drawn several free-body diagrams in previous worked examples. **Figure 6.2**(c) shows a free-body diagram for the system of interest. Note that no internal forces are shown in a free-body diagram.

Once a free-body diagram is drawn, we apply Newton's second law. This is done in **Figure 6.2**(d) for a particular situation. In general, once external forces are clearly identified in free-body diagrams, it should be a straightforward task to put them into equation form and solve for the unknown, as done in all previous examples. If the problem is one-dimensional—that is, if all forces are parallel—then the forces can be handled algebraically. If the problem is two-dimensional, then it must be broken down into a pair of one-dimensional problems. We do this by projecting the force vectors onto a set of axes chosen for convenience. As seen in previous examples, the choice of axes can simplify the problem. For example, when an incline is involved, a set of axes with one axis parallel to the incline and one perpendicular to it is most convenient. It is almost always convenient to make one axis parallel to the direction of motion, if this is known. Generally, just write Newton's second law in components along the different directions. Then, you have the following equations:

$$\sum F_x = ma_x, \quad \sum F_y = ma_y.$$

(If, for example, the system is accelerating horizontally, then you can then set $a_y = 0.$) We need this information to determine unknown forces acting on a system.

As always, we must check the solution. In some cases, it is easy to tell whether the solution is reasonable. For example, it is reasonable to find that friction causes an object to slide down an incline more slowly than when no friction exists. In practice, intuition develops gradually through problem solving; with experience, it becomes progressively easier to judge whether an answer is reasonable. Another way to check a solution is to check the units. If we are solving for force and end up with units of millimeters per second, then we have made a mistake.

There are many interesting applications of Newton's laws of motion, a few more of which are presented in this section. These serve also to illustrate some further subtleties of physics and to help build problem-solving skills. We look first at problems involving particle equilibrium, which make use of Newton's first law, and then consider particle acceleration, which involves Newton's second law.

Particle Equilibrium

Recall that a particle in equilibrium is one for which the external forces are balanced. Static equilibrium involves objects at rest, and dynamic equilibrium involves objects in motion without acceleration, but it is important to remember that these conditions are relative. For example, an object may be at rest when viewed from our frame of reference, but the same object would appear to be in motion when viewed by someone moving at a constant velocity. We now make use of the knowledge attained in **Newton's Laws of Motion**, regarding the different types of forces and the use of free-body diagrams, to solve additional problems in particle equilibrium.

Example 6.1

Different Tensions at Different Angles

Consider the traffic light (mass of 15.0 kg) suspended from two wires as shown in **Figure 6.3**. Find the tension in each wire, neglecting the masses of the wires.

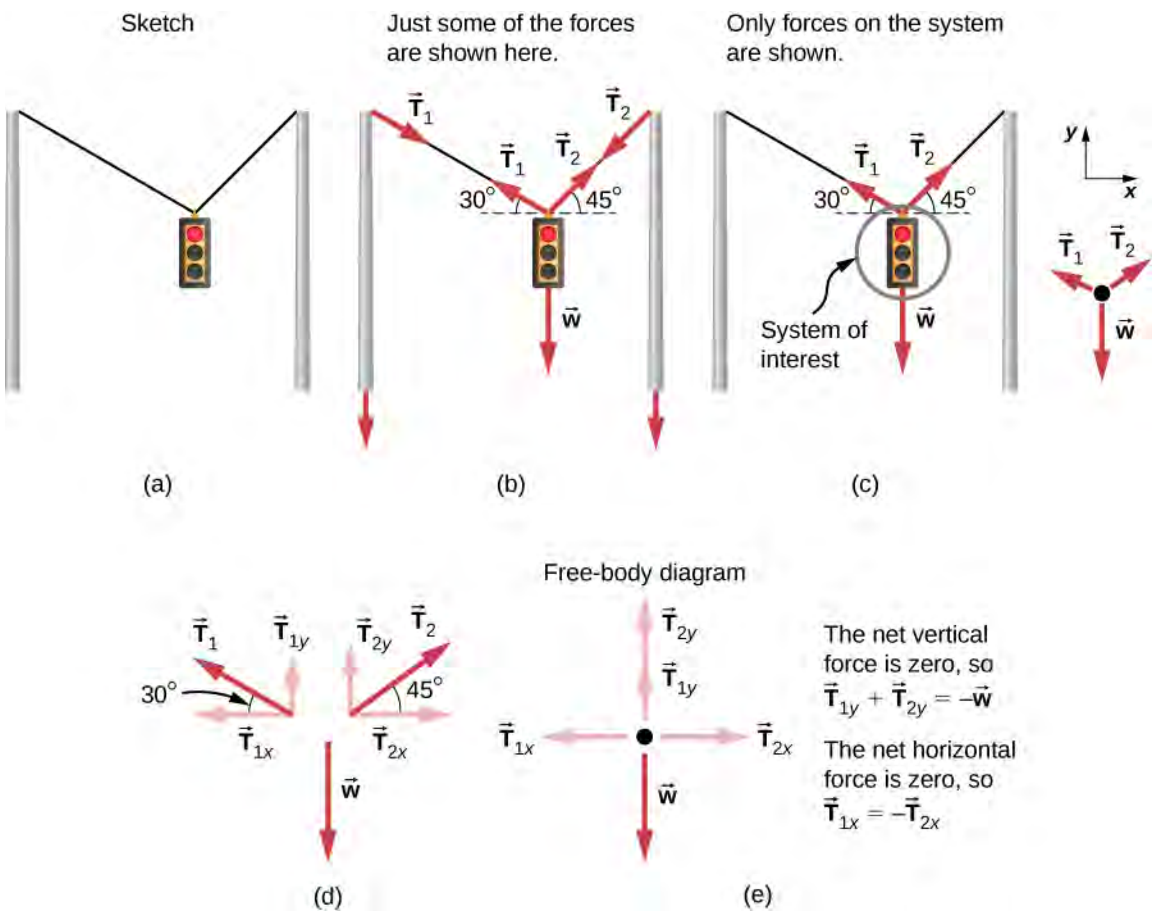


Figure 6.3 A traffic light is suspended from two wires. (b) Some of the forces involved. (c) Only forces acting on the system are shown here. The free-body diagram for the traffic light is also shown. (d) The forces projected onto vertical (y) and horizontal (x) axes. The horizontal components of the tensions must cancel, and the sum of the vertical components of the tensions must equal the weight of the traffic light. (e) The free-body diagram shows the vertical and horizontal forces acting on the traffic light.

Strategy

The system of interest is the traffic light, and its free-body diagram is shown in **Figure 6.3**(c). The three forces involved are not parallel, and so they must be projected onto a coordinate system. The most convenient coordinate system has one axis vertical and one horizontal, and the vector projections on it are shown in **Figure 6.3**(d). There are two unknowns in this problem (T_1 and T_2), so two equations are needed to find them. These two equations come from applying Newton's second law along the vertical and horizontal axes, noting that the net external force is zero along each axis because acceleration is zero.

Solution

First consider the horizontal or *x*-axis:

$$F_{\text{net}\,x} = T_{2x} - T_{1x} = 0.$$

Thus, as you might expect,

$$T_{1x} = T_{2x}.$$

This gives us the following relationship:

$$T_1 \cos 30° = T_2 \cos 45°.$$

Thus,

$$T_2 = 1.225T_1.$$

Note that T_1 and T_2 are not equal in this case because the angles on either side are not equal. It is reasonable that T_2 ends up being greater than T_1 because it is exerted more vertically than T_1.

Now consider the force components along the vertical or *y*-axis:

$$F_{\text{net}\,y} = T_{1y} + T_{2y} - w = 0.$$

This implies

$$T_{1y} + T_{2y} = w.$$

Substituting the expressions for the vertical components gives

$$T_1 \sin 30° + T_2 \sin 45° = w.$$

There are two unknowns in this equation, but substituting the expression for T_2 in terms of T_1 reduces this to one equation with one unknown:

$$T_1(0.500) + (1.225T_1)(0.707) = w = mg,$$

which yields

$$1.366T_1 = (15.0\,\text{kg})(9.80\,\text{m/s}^2).$$

Solving this last equation gives the magnitude of T_1 to be

$$T_1 = 108\,\text{N}.$$

Finally, we find the magnitude of T_2 by using the relationship between them, $T_2 = 1.225T_1$, found above. Thus we obtain

$$T_2 = 132\,\text{N}.$$

Significance

Both tensions would be larger if both wires were more horizontal, and they will be equal if and only if the angles on either side are the same (as they were in the earlier example of a tightrope walker in **Newton's Laws of Motion**.

Particle Acceleration

We have given a variety of examples of particles in equilibrium. We now turn our attention to particle acceleration problems, which are the result of a nonzero net force. Refer again to the steps given at the beginning of this section, and notice how they are applied to the following examples.

Example 6.2

Drag Force on a Barge

Two tugboats push on a barge at different angles (**Figure 6.4**). The first tugboat exerts a force of 2.7×10^5 N in the *x*-direction, and the second tugboat exerts a force of 3.6×10^5 N in the *y*-direction. The mass of the barge is 5.0×10^6 kg and its acceleration is observed to be 7.5×10^{-2} m/s^2 in the direction shown. What is the drag force of the water on the barge resisting the motion? (*Note:* Drag force is a frictional force exerted by fluids, such as air or water. The drag force opposes the motion of the object. Since the barge is flat bottomed, we can assume that the drag force is in the direction opposite of motion of the barge.)

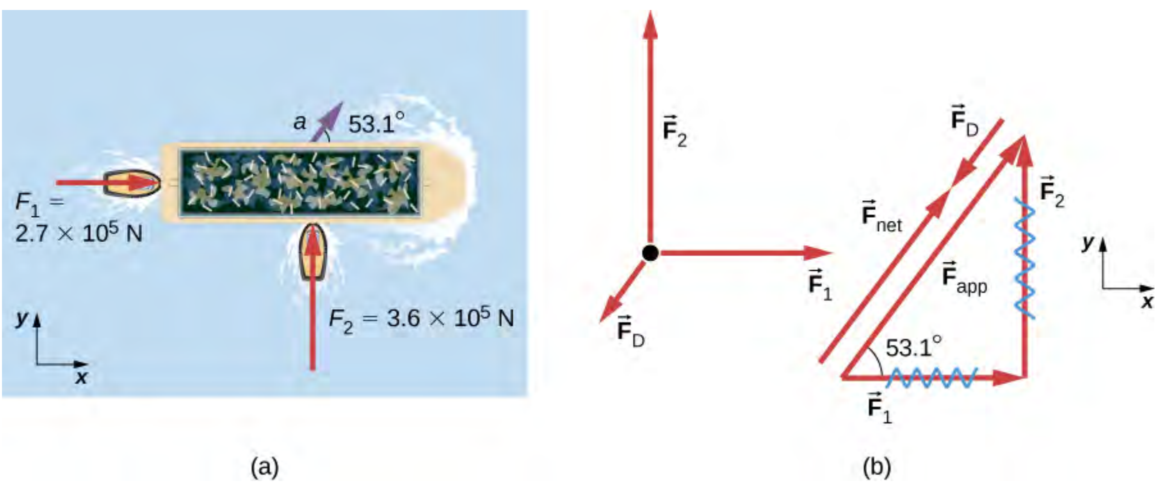


Figure 6.4 (a) A view from above of two tugboats pushing on a barge. (b) The free-body diagram for the ship contains only forces acting in the plane of the water. It omits the two vertical forces—the weight of the barge and the buoyant force of the water supporting it cancel and are not shown. Note that $\vec{F}_{app}$ is the total applied force of the tugboats.

Strategy

The directions and magnitudes of acceleration and the applied forces are given in **Figure 6.4**(a). We define the total force of the tugboats on the barge as $\vec{F}_{app}$ so that

$$\vec{F}_{app} = \vec{F}_1 + \vec{F}_2.$$

The drag of the water $\vec{F}_D$ is in the direction opposite to the direction of motion of the boat; this force thus works against $\vec{F}_{app}$, as shown in the free-body diagram in **Figure 6.4**(b). The system of interest here is the barge, since the forces on it are given as well as its acceleration. Because the applied forces are perpendicular, the *x*- and *y*-axes are in the same direction as $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. The problem quickly becomes a one-dimensional problem along the direction of $\vec{\mathbf{F}}_{\text{app}}$, since friction is in the direction opposite to $\vec{\mathbf{F}}_{\text{app}}$. Our strategy is to find the magnitude and direction of the net applied force $\vec{\mathbf{F}}_{\text{app}}$ and then apply Newton's second law to solve for the drag force $\vec{\mathbf{F}}_{\text{D}}$.

Solution

Since F_x and F_y are perpendicular, we can find the magnitude and direction of $\vec{\mathbf{F}}_{\text{app}}$ directly. First, the resultant magnitude is given by the Pythagorean theorem:

$$F_{\text{app}} = \sqrt{F_1^2 + F_2^2} = \sqrt{(2.7 \times 10^5 \text{N})^2 + (3.6 \times 10^5 \text{ N})^2} = 4.5 \times 10^5 \text{ N}.$$

The angle is given by

$$\theta = \tan^{-1}\left(\frac{F_2}{F_1}\right) = \tan^{-1}\left(\frac{3.6 \times 10^5 \text{ N}}{2.7 \times 10^5 \text{ N}}\right) = 53.1°.$$

From Newton's first law, we know this is the same direction as the acceleration. We also know that $\vec{\mathbf{F}}_{\text{D}}$ is in the opposite direction of $\vec{\mathbf{F}}_{\text{app}}$, since it acts to slow down the acceleration. Therefore, the net external force is in the same direction as $\vec{\mathbf{F}}_{\text{app}}$, but its magnitude is slightly less than $\vec{\mathbf{F}}_{\text{app}}$. The problem is now one-dimensional. From the free-body diagram, we can see that

$$F_{\text{net}} = F_{\text{app}} - F_{\text{D}}.$$

However, Newton's second law states that

$$F_{\text{net}} = ma.$$

Thus,

$$F_{\text{app}} - F_{\text{D}} = ma.$$

This can be solved for the magnitude of the drag force of the water F_{D} in terms of known quantities:

$$F_{\text{D}} = F_{\text{app}} - ma.$$

Substituting known values gives

$$F_{\text{D}} = \left(4.5 \times 10^5 \text{ N}\right) - \left(5.0 \times 10^6 \text{ kg}\right)\left(7.5 \times 10^{-2} \text{ m/s}^2\right) = 7.5 \times 10^4 \text{ N}.$$

The direction of $\vec{\mathbf{F}}_{\text{D}}$ has already been determined to be in the direction opposite to $\vec{\mathbf{F}}_{\text{app}}$, or at an angle of $53°$ south of west.

Significance

The numbers used in this example are reasonable for a moderately large barge. It is certainly difficult to obtain larger accelerations with tugboats, and small speeds are desirable to avoid running the barge into the docks. Drag is relatively small for a well-designed hull at low speeds, consistent with the answer to this example, where F_{D} is less than 1/600th of the weight of the ship.

In **Newton's Laws of Motion**, we discussed the normal force, which is a contact force that acts normal to the surface so that an object does not have an acceleration perpendicular to the surface. The bathroom scale is an excellent example of a normal force acting on a body. It provides a quantitative reading of how much it must push upward to support the weight of an object. But can you predict what you would see on the dial of a bathroom scale if you stood on it during an elevator ride?

Will you see a value greater than your weight when the elevator starts up? What about when the elevator moves upward at a constant speed? Take a guess before reading the next example.

Example 6.3

What Does the Bathroom Scale Read in an Elevator?

Figure 6.5 shows a 75.0-kg man (weight of about 165 lb.) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of 1.20 m/s^2, and (b) if the elevator moves upward at a constant speed of 1 m/s.

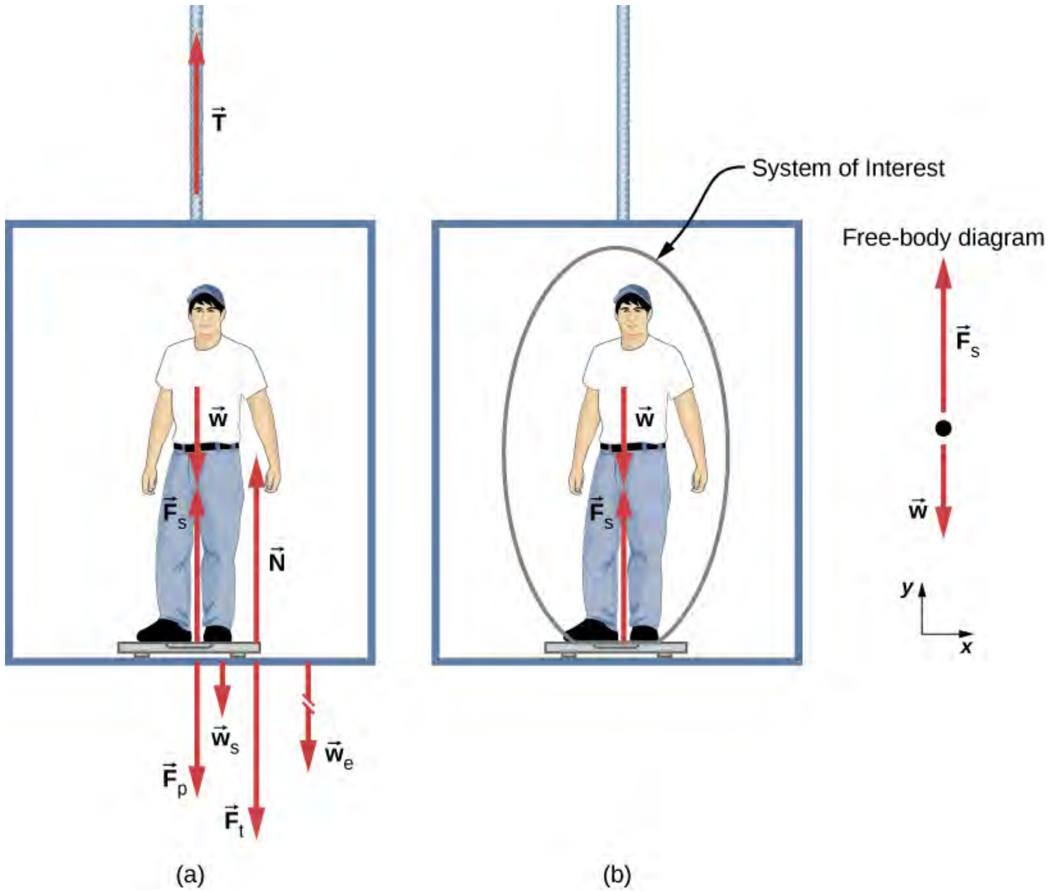


Figure 6.5 (a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\vec{T}$ is the tension in the supporting cable, $\vec{w}$ is the weight of the person, $\vec{w}_s$ is the weight of the scale, $\vec{w}_e$ is the weight of the elevator, $\vec{F}_s$ is the force of the scale on the person, $\vec{F}_p$ is the force of the person on the scale, $\vec{F}_t$ is the force of the scale on the floor of the elevator, and $\vec{N}$ is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting *on* the designated system of interest—the person—and is the diagram we use for the solution of the problem.

Strategy

If the scale at rest is accurate, its reading equals $\vec{F}_p$, the magnitude of the force the person exerts downward on it. Figure 6.5(a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn, as in **Figure 6.5**(b). Analysis of the free-body diagram using Newton’s laws can produce answers to both **Figure 6.5**(a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\vec{\mathbf{w}}$ and the upward force of the scale $\vec{\mathbf{F}}_s$. According to Newton’s third law, $\vec{\mathbf{F}}_p$ and $\vec{\mathbf{F}}_s$ are equal in magnitude and opposite in direction, so that we need to find F_s in order to find what the scale reads. We can do this, as usual, by applying Newton’s second law,

$$\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}}.$$

From the free-body diagram, we see that $\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_s - \vec{\mathbf{w}}$, so we have

$$F_s - w = ma.$$

Solving for F_s gives us an equation with only one unknown:

$$F_s = ma + w,$$

or, because $w = mg$, simply

$$F_s = ma + mg.$$

No assumptions were made about the acceleration, so this solution should be valid for a variety of accelerations in addition to those in this situation. (*Note:* We are considering the case when the elevator is accelerating upward. If the elevator is accelerating downward, Newton’s second law becomes $F_s - w = -ma.$)

Solution

a. We have $a = 1.20\ \text{m/s}^2$, so that

$$F_s = (75.0\ \text{kg})(9.80\ \text{m/s}^2) + (75.0\ \text{kg})(1.20\ \text{m/s}^2)$$

yielding

$$F_s = 825\ \text{N}.$$

b. Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$ and $\Delta v = 0$. Thus,

$$F_s = ma + mg = 0 + mg$$

or

$$F_s = (75.0\ \text{kg})(9.80\ \text{m/s}^2),$$

which gives

$$F_s = 735\ \text{N}.$$

Significance

The scale reading in **Figure 6.5**(a) is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

$$F_{net} = ma = 0 = F_s - w$$
$$F_s = w = mg$$
$$F_s = (75.0\ \text{kg})(9.80\ \text{m/s}^2) = 735\ \text{N}.$$

Thus, the scale reading in the elevator is greater than his 735-N (165-lb.) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward.

Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators. In **Figure 6.5**(b), the scale reading is 735 N, which equals the person's weight. This is the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

6.1 Check Your Understanding Now calculate the scale reading when the elevator accelerates downward at a rate of $1.20\ \text{m/s}^2$.

The solution to the previous example also applies to an elevator accelerating downward, as mentioned. When an elevator accelerates downward, *a* is negative, and the scale reading is *less* than the weight of the person. If a constant downward velocity is reached, the scale reading again becomes equal to the person's weight. If the elevator is in free fall and accelerating downward at *g*, then the scale reading is zero and the person appears to be weightless.

Example 6.4

Two Attached Blocks

Figure 6.6 shows a block of mass m_1 on a frictionless, horizontal surface. It is pulled by a light string that passes over a frictionless and massless pulley. The other end of the string is connected to a block of mass m_2. Find the acceleration of the blocks and the tension in the string in terms of m_1, m_2, and g.

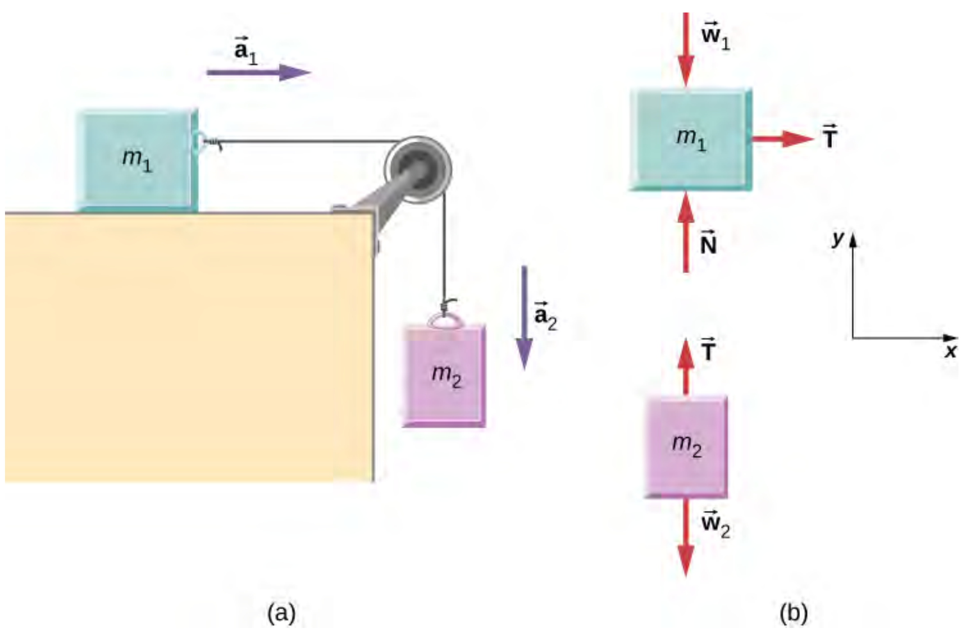


Figure 6.6 (a) Block 1 is connected by a light string to block 2. (b) The free-body diagrams of the blocks.

Strategy

We draw a free-body diagram for each mass separately, as shown in **Figure 6.6**. Then we analyze each one to find the required unknowns. The forces on block 1 are the gravitational force, the contact force of the surface, and the tension in the string. Block 2 is subjected to the gravitational force and the string tension. Newton's second law applies to each, so we write two vector equations:

For block 1: $\vec{\mathbf{T}} + \vec{\mathbf{w}}_1 + \vec{\mathbf{N}} = m_1 \vec{\mathbf{a}}_1$

For block 2: $\vec{\mathbf{T}} + \vec{\mathbf{w}}_2 = m_2 \vec{\mathbf{a}}_2.$

Notice that $\vec{\mathbf{T}}$ is the same for both blocks. Since the string and the pulley have negligible mass, and since there is no friction in the pulley, the tension is the same throughout the string. We can now write component equations for each block. All forces are either horizontal or vertical, so we can use the same horizontal/vertical coordinate system for both objects

Solution

The component equations follow from the vector equations above. We see that block 1 has the vertical forces balanced, so we ignore them and write an equation relating the *x*-components. There are no horizontal forces on block 2, so only the *y*-equation is written. We obtain these results:

$$\begin{array}{ll} \textbf{Block 1} & \textbf{Block 2} \\ \sum F_x = ma_x & \sum F_y = ma_y \\ T_x = m_1 a_{1x} & T_y - m_2 g = m_2 a_{2y}. \end{array}$$

When block 1 moves to the right, block 2 travels an equal distance downward; thus, $a_{1x} = -a_{2y}$. Writing the common acceleration of the blocks as $a = a_{1x} = -a_{2y}$, we now have

$$T = m_1 a$$

and

$$T - m_2 g = -m_2 a.$$

From these two equations, we can express *a* and *T* in terms of the masses m_1 and m_2, and g :

$$a = \frac{m_2}{m_1 + m_2} g$$

and

$$T = \frac{m_1 m_2}{m_1 + m_2} g.$$

Significance

Notice that the tension in the string is *less* than the weight of the block hanging from the end of it. A common error in problems like this is to set $T = m_2 g$. You can see from the free-body diagram of block 2 that cannot be correct if the block is accelerating.

6.2 Check Your Understanding Calculate the acceleration of the system, and the tension in the string, when the masses are $m_1 = 5.00\,\text{kg}$ and $m_2 = 3.00\,\text{kg}$.

Example 6.5

Atwood Machine

A classic problem in physics, similar to the one we just solved, is that of the Atwood machine, which consists of a rope running over a pulley, with two objects of different mass attached. It is particularly useful in understanding the connection between force and motion. In **Figure 6.7**, $m_1 = 2.00\,\text{kg}$ and $m_2 = 4.00\,\text{kg}$. Consider the pulley to be frictionless. (a) If m_2 is released, what will its acceleration be? (b) What is the tension in the string?

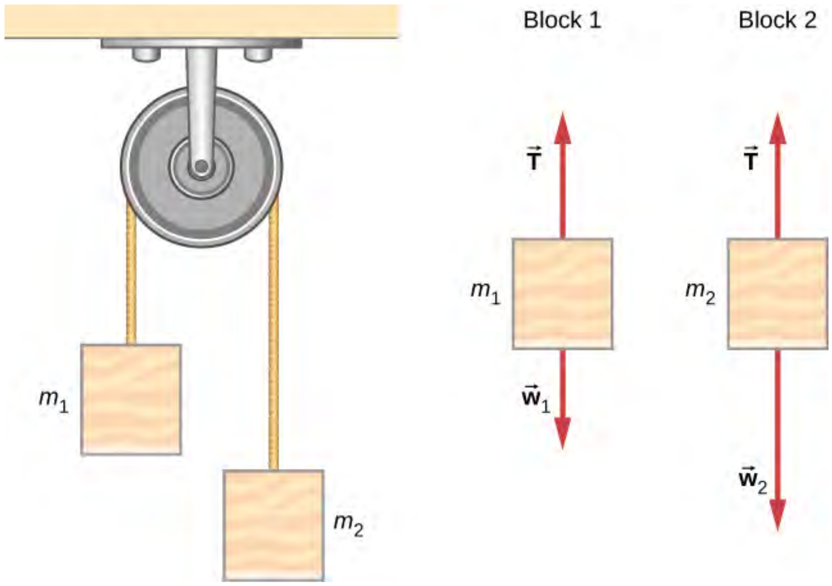


Figure 6.7 An Atwood machine and free-body diagrams for each of the two blocks.

Strategy

We draw a free-body diagram for each mass separately, as shown in the figure. Then we analyze each diagram to find the required unknowns. This may involve the solution of simultaneous equations. It is also important to note the similarity with the previous example. As block 2 accelerates with acceleration a_2 in the downward direction, block 1 accelerates upward with acceleration a_1. Thus, $a = a_1 = -a_2$.

Solution

a. We have

$$\text{For } m_1, \ \sum F_y = T - m_1 g = m_1 a. \qquad \text{For } m_2, \ \sum F_y = T - m_2 g = -m_2 a.$$

(The negative sign in front of $m_2 a$ indicates that m_2 accelerates downward; both blocks accelerate at the same rate, but in opposite directions.) Solve the two equations simultaneously (subtract them) and the result is

$$(m_2 - m_1)g = (m_1 + m_2)a.$$

Solving for a:

$$a = \frac{m_2 - m_1}{m_1 + m_2} g = \frac{4\text{ kg} - 2\text{ kg}}{4\text{ kg} + 2\text{ kg}}(9.8\text{ m/s}^2) = 3.27\text{ m/s}^2.$$

b. Observing the first block, we see that

$$T - m_1 g = m_1 a$$
$$T = m_1(g + a) = (2\text{ kg})(9.8\text{ m/s}^2 + 3.27\text{ m/s}^2) = 26.1\text{ N}.$$

Significance

The result for the acceleration given in the solution can be interpreted as the ratio of the unbalanced force on the system, $(m_2 - m_1)g$, to the total mass of the system, $m_1 + m_2$. We can also use the Atwood machine to measure local gravitational field strength.

6.3 Check Your Understanding Determine a general formula in terms of m_1, m_2 and g for calculating the tension in the string for the Atwood machine shown above.

Newton’s Laws of Motion and Kinematics

Physics is most interesting and most powerful when applied to general situations that involve more than a narrow set of physical principles. Newton’s laws of motion can also be integrated with other concepts that have been discussed previously in this text to solve problems of motion. For example, forces produce accelerations, a topic of kinematics, and hence the relevance of earlier chapters.

When approaching problems that involve various types of forces, acceleration, velocity, and/or position, listing the givens and the quantities to be calculated will allow you to identify the principles involved. Then, you can refer to the chapters that deal with a particular topic and solve the problem using strategies outlined in the text. The following worked example illustrates how the problem-solving strategy given earlier in this chapter, as well as strategies presented in other chapters, is applied to an integrated concept problem.

Example 6.6

What Force Must a Soccer Player Exert to Reach Top Speed?

A soccer player starts at rest and accelerates forward, reaching a velocity of 8.00 m/s in 2.50 s. (a) What is her average acceleration? (b) What average force does the ground exert forward on the runner so that she achieves this acceleration? The player’s mass is 70.0 kg, and air resistance is negligible.

Strategy

To find the answers to this problem, we use the problem-solving strategy given earlier in this chapter. The solutions to each part of the example illustrate how to apply specific problem-solving steps. In this case, we do not need to use all of the steps. We simply identify the physical principles, and thus the knowns and unknowns; apply Newton’s second law; and check to see whether the answer is reasonable.

Solution

a. We are given the initial and final velocities (zero and 8.00 m/s forward); thus, the change in velocity is $\Delta v = 8.00\,\text{m/s}$. We are given the elapsed time, so $\Delta t = 2.50\,\text{s}$. The unknown is acceleration, which can be found from its definition:

$$a = \frac{\Delta v}{\Delta t}.$$

Substituting the known values yields

$$a = \frac{8.00\,\text{m/s}}{2.50\,\text{s}} = 3.20\,\text{m/s}^2.$$

b. Here we are asked to find the average force the ground exerts on the runner to produce this acceleration. (Remember that we are dealing with the force or forces acting on the object of interest.) This is the reaction force to that exerted by the player backward against the ground, by Newton’s third law. Neglecting air resistance, this would be equal in magnitude to the net external force on the player, since this force causes her acceleration. Since we now know the player’s acceleration and are given her mass, we can use Newton’s second law to find the force exerted. That is,

$$F_{\text{net}} = ma.$$

Substituting the known values of m and a gives

$$F_{\text{net}} = (70.0\,\text{kg})(3.20\,\text{m/s}^2) = 224\,\text{N}.$$

This is a reasonable result: The acceleration is attainable for an athlete in good condition. The force is about 50 pounds, a reasonable average force.

Significance

This example illustrates how to apply problem-solving strategies to situations that include topics from different chapters. The first step is to identify the physical principles, the knowns, and the unknowns involved in the problem. The second step is to solve for the unknown, in this case using Newton's second law. Finally, we check our answer to ensure it is reasonable. These techniques for integrated concept problems will be useful in applications of physics outside of a physics course, such as in your profession, in other science disciplines, and in everyday life.

6.4 Check Your Understanding The soccer player stops after completing the play described above, but now notices that the ball is in position to be stolen. If she now experiences a force of 126 N to attempt to steal the ball, which is 2.00 m away from her, how long will it take her to get to the ball?

Example 6.7

What Force Acts on a Model Helicopter?

A 1.50-kg model helicopter has a velocity of $5.00\,\hat{\mathbf{j}}$ m/s at $t = 0.$ It is accelerated at a constant rate for two seconds (2.00 s) after which it has a velocity of $\left(6.00\,\hat{\mathbf{i}} + 12.00\,\hat{\mathbf{j}}\right)$m/s. What is the magnitude of the resultant force acting on the helicopter during this time interval?

Strategy

We can easily set up a coordinate system in which the *x*-axis ($\hat{\mathbf{i}}$ direction) is horizontal, and the *y*-axis ($\hat{\mathbf{j}}$ direction) is vertical. We know that $\Delta t = 2.00s$ and $(6.00\,\hat{\mathbf{i}} + 12.00\,\hat{\mathbf{j}}\text{ m/s}) - (5.00\,\hat{\mathbf{j}}\text{ m/s}).$ From this, we can calculate the acceleration by the definition; we can then apply Newton's second law.

Solution

We have

$$a = \frac{\Delta v}{\Delta t} = \frac{(6.00\,\hat{\mathbf{i}} + 12.00\,\hat{\mathbf{j}}\text{ m/s}) - (5.00\,\hat{\mathbf{j}}\text{ m/s})}{2.00\text{ s}} = 3.00\,\hat{\mathbf{i}} + 3.50\,\hat{\mathbf{j}}\text{ m/s}^2$$

$$\sum \vec{\mathbf{F}} = m\,\vec{\mathbf{a}} = (1.50\text{ kg})(3.00\,\hat{\mathbf{i}} + 3.50\,\hat{\mathbf{j}}\text{ m/s}^2) = 4.50\,\hat{\mathbf{i}} + 5.25\,\hat{\mathbf{j}}\text{ N}.$$

The magnitude of the force is now easily found:

$$F = \sqrt{(4.50\text{ N})^2 + (5.25\text{ N})^2} = 6.91\text{ N}.$$

Significance

The original problem was stated in terms of $\hat{\mathbf{i}} - \hat{\mathbf{j}}$ vector components, so we used vector methods. Compare this example with the previous example.

6.5 Check Your Understanding Find the direction of the resultant for the 1.50-kg model helicopter.

Example 6.8

Baggage Tractor

Figure 6.8(a) shows a baggage tractor pulling luggage carts from an airplane. The tractor has mass 650.0 kg, while cart A has mass 250.0 kg and cart B has mass 150.0 kg. The driving force acting for a brief period of time accelerates the system from rest and acts for 3.00 s. (a) If this driving force is given by $F = (820.0t)\text{ N}$, find the speed after 3.00 seconds. (b) What is the horizontal force acting on the connecting cable between the tractor and cart A at this instant?

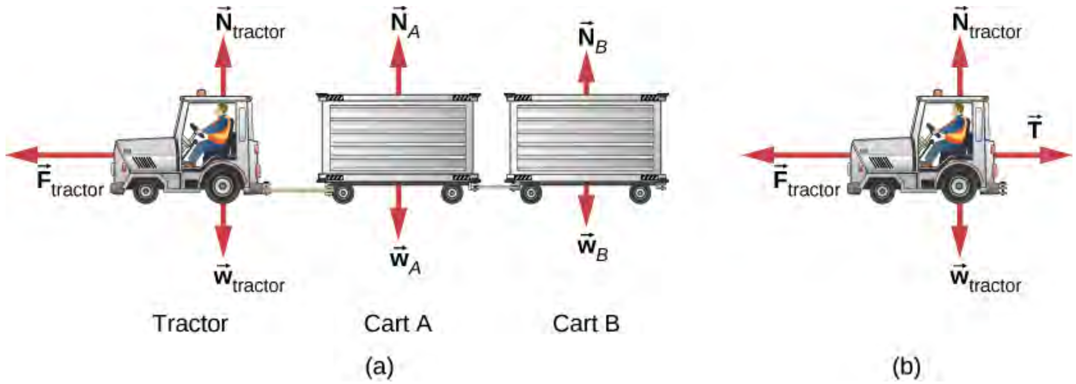


Figure 6.8 (a) A free-body diagram is shown, which indicates all the external forces on the system consisting of the tractor and baggage carts for carrying airline luggage. (b) A free-body diagram of the tractor only is shown isolated in order to calculate the tension in the cable to the carts.

Strategy

A free-body diagram shows the driving force of the tractor, which gives the system its acceleration. We only need to consider motion in the horizontal direction. The vertical forces balance each other and it is not necessary to consider them. For part b, we make use of a free-body diagram of the tractor alone to determine the force between it and cart A. This exposes the coupling force $\vec{\mathbf{T}}$, which is our objective.

Solution

a. $\sum F_x = m_{\text{system}} a_x$ and $\sum F_x = 820.0t,$ so

$$\begin{aligned} 820.0t &= (650.0 + 250.0 + 150.0)a \\ a &= 0.7809t. \end{aligned}$$

Since acceleration is a function of time, we can determine the velocity of the tractor by using $a = \frac{dv}{dt}$ with the initial condition that $v_0 = 0$ at $t = 0.$ We integrate from $t = 0$ to $t = 3$:

$$dv = adt, \quad \int_0^3 dv = \int_0^{3.00} adt = \int_0^{3.00} 0.7809tdt, \quad v = 0.3905t^2\Big]_0^{3.00} = 3.51\text{ m/s}.$$

b. Refer to the free-body diagram in **Figure 6.8**(b).

$$\begin{aligned} \sum F_x &= m_{\text{tractor}} a_x \\ 820.0t - T &= m_{\text{tractor}}(0.7805)t \\ (820.0)(3.00) - T &= (650.0)(0.7805)(3.00) \\ T &= 938\text{ N}. \end{aligned}$$

Significance

Since the force varies with time, we must use calculus to solve this problem. Notice how the total mass of the system was important in solving Figure 6.8(a), whereas only the mass of the truck (since it supplied the force) was of use in Figure 6.8(b).

Recall that $v = \frac{ds}{dt}$ and $a = \frac{dv}{dt}$. If acceleration is a function of time, we can use the calculus forms developed in Motion Along a Straight Line, as shown in this example. However, sometimes acceleration is a function of displacement. In this case, we can derive an important result from these calculus relations. Solving for dt in each, we have $dt = \frac{ds}{v}$ and $dt = \frac{dv}{a}$. Now, equating these expressions, we have $\frac{ds}{v} = \frac{dv}{a}$. We can rearrange this to obtain $a\,ds = v\,dv$.

Example 6.9

Motion of a Projectile Fired Vertically

A 10.0-kg mortar shell is fired vertically upward from the ground, with an initial velocity of 50.0 m/s (see Figure 6.9). Determine the maximum height it will travel if atmospheric resistance is measured as $F_D = (0.0100v^2)$ N, where v is the speed at any instant.

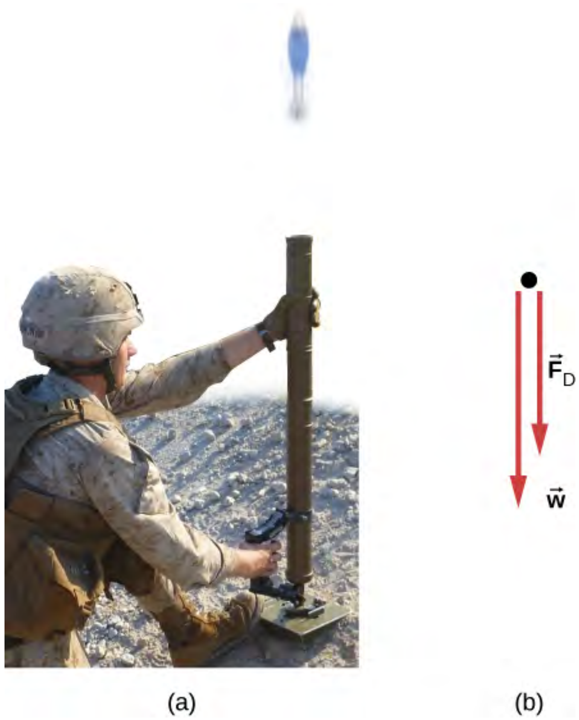


Figure 6.9 (a) The mortar fires a shell straight up; we consider the friction force provided by the air. (b) A free-body diagram is shown which indicates all the forces on the mortar shell.

Strategy

The known force on the mortar shell can be related to its acceleration using the equations of motion. Kinematics can then be used to relate the mortar shell's acceleration to its position.

Solution

Initially, $y_0 = 0$ and $v_0 = 50.0\,\text{m/s}$. At the maximum height $y = h$, $v = 0$. The free-body diagram shows F_D to act downward, because it slows the upward motion of the mortar shell. Thus, we can write

$$\sum F_y = ma_y$$

$$\begin{aligned} -F_\text{D} - w &= ma_y \\ -0.0100v^2 - 98.0 &= 10.0a \\ a &= -0.00100v^2 - 9.80. \end{aligned}$$

The acceleration depends on v and is therefore variable. Since $a = f(v)$, we can relate a to v using the rearrangement described above,

$$a\,ds = v\,dv.$$

We replace *ds* with *dy* because we are dealing with the vertical direction,

$$ady = vdv, \quad (-0.00100v^2 - 9.80)dy = vdv.$$

We now separate the variables (v's and dv's on one side; dy on the other):

$$\int_0^h dy = \int_{50.0}^{0} \frac{vdv}{(-0.00100v^2 - 9.80)}$$

$$\int_0^h dy = -\int_{50.0}^{0} \frac{vdv}{(0.00100v^2 + 9.80)} = (-5 \times 10^3)\ln(0.00100v^2 + 9.80)\Big|_{50.0}^{0}.$$

Thus, $h = 114\,\text{m}$.

Significance

Notice the need to apply calculus since the force is not constant, which also means that acceleration is not constant. To make matters worse, the force depends on v (not t), and so we must use the trick explained prior to the example. The answer for the height indicates a lower elevation if there were air resistance. We will deal with the effects of air resistance and other drag forces in greater detail in **Drag Force and Terminal Speed**.

6.6 Check Your Understanding If atmospheric resistance is neglected, find the maximum height for the mortar shell. Is calculus required for this solution?

Explore the forces at work in this **simulation (https://openstaxcollege.org/l/21forcesatwork)** when you try to push a filing cabinet. Create an applied force and see the resulting frictional force and total force acting on the cabinet. Charts show the forces, position, velocity, and acceleration vs. time. View a free-body diagram of all the forces (including gravitational and normal forces).

6.2 | Friction

Learning Objectives

By the end of the section, you will be able to:

- Describe the general characteristics of friction
- List the various types of friction
- Calculate the magnitude of static and kinetic friction, and use these in problems involving Newton’s laws of motion

When a body is in motion, it has resistance because the body interacts with its surroundings. This resistance is a force of friction. Friction opposes relative motion between systems in contact but also allows us to move, a concept that becomes obvious if you try to walk on ice. Friction is a common yet complex force, and its behavior still not completely understood. Still, it is possible to understand the circumstances in which it behaves.

Static and Kinetic Friction

The basic definition of **friction** is relatively simple to state.

Friction

Friction is a force that opposes relative motion between systems in contact.

There are several forms of friction. One of the simpler characteristics of sliding friction is that it is parallel to the contact surfaces between systems and is always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction. For example, friction slows a hockey puck sliding on ice. When objects are stationary, static friction can act between them; the static friction is usually greater than the kinetic friction between two objects.

Static and Kinetic Friction

If two systems are in contact and stationary relative to one another, then the friction between them is called **static friction**. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you might push very hard on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. If you finally push hard enough, the crate seems to slip suddenly and starts to move. Now static friction gives way to kinetic friction. Once in motion, it is easier to keep it in motion than it was to get it started, indicating that the kinetic frictional force is less than the static frictional force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it easier to get the crate started and keep it going (as you might expect).

Figure 6.10 is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. Thus, when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, breaking off the points, or both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explains the dependence of friction on the nature of the substances. For example, rubber-soled shoes slip less than those with leather soles. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.

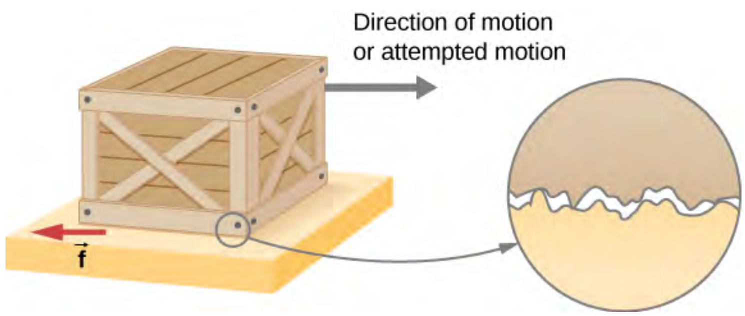


Figure 6.10 Frictional forces, such as $\vec{\mathbf{f}}$, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view. For the object to move, it must rise to where the peaks of the top surface can skip along the bottom surface. Thus, a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. (In fact, perfectly smooth, clean surfaces of similar materials would adhere, forming a bond called a "cold weld.")

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for situations involving motion (kinetic friction). What follows is an approximate empirical (experimentally determined) model only. These equations for static and kinetic friction are not vector equations.

Magnitude of Static Friction

The magnitude of static friction f_s is

$$f_s \leq \mu_s N, \tag{6.1}$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a maximum value of $\mu_s N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_s(\text{max})$, the object moves. Thus,

$$f_s(\text{max}) = \mu_s N.$$

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_k is given by

$$f_k = \mu_k N, \tag{6.2}$$

where μ_k is the coefficient of kinetic friction.

A system in which $f_k = \mu_k N$ is described as a system in which *friction behaves simply*. The transition from static friction to kinetic friction is illustrated in **Figure 6.11**.

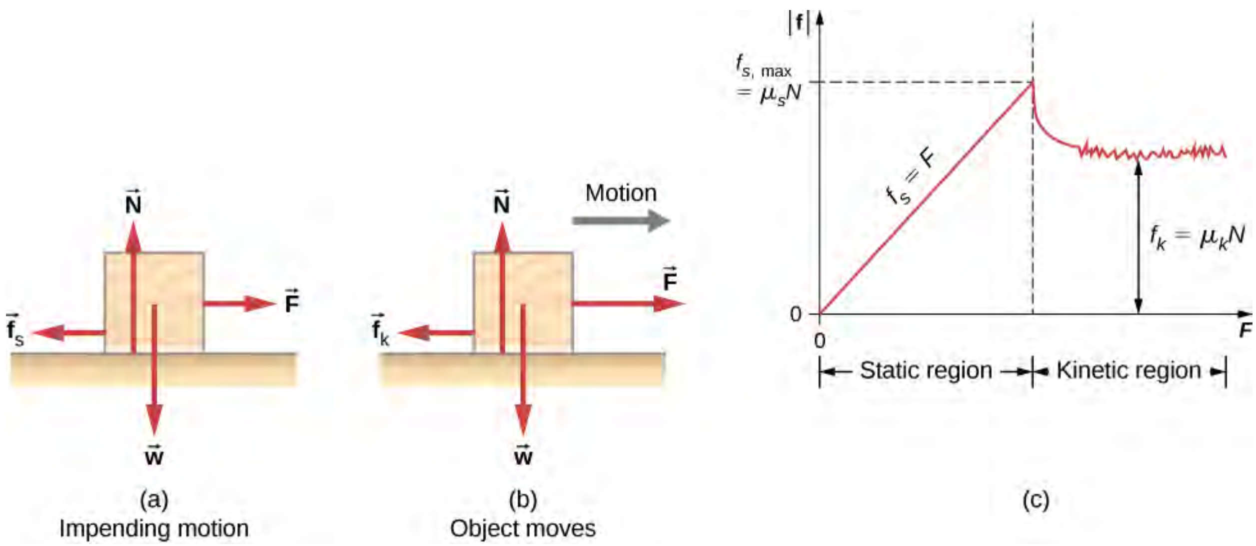


Figure 6.11 (a) The force of friction $\vec{f}$ between the block and the rough surface opposes the direction of the applied force $\vec{F}$. The magnitude of the static friction balances that of the applied force. This is shown in the left side of the graph in (c). (b) At some point, the magnitude of the applied force is greater than the force of kinetic friction, and the block moves to the right. This is shown in the right side of the graph. (c) The graph of the frictional force versus the applied force; note that $f_s(\text{max}) > f_k$. This means that $\mu_s > \mu_k$.

As you can see in **Table 6.1**, the coefficients of kinetic friction are less than their static counterparts. The approximate values of μ are stated to only one or two digits to indicate the approximate description of friction given by the preceding two equations.

System	Static Friction μ_s	Kinetic Friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.5-0.7	0.3-0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

Table 6.1 Approximate Coefficients of Static and Kinetic Friction

Equation 6.1 and **Equation 6.2** include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force is equal to its weight,

$$w = mg = (100\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 980\ \text{N},$$

perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than

$$f_s(\text{max}) = \mu_s N = (0.45)(980\ \text{N}) = 440\ \text{N}$$

to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only

$$f_k = \mu_k N = (0.30)(980\ \text{N}) = 290\ \text{N}$$

keeps it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unitless quantity with a magnitude usually between 0 and 1.0. The actual value depends on the two surfaces that are in contact.

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost-glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint (**Figure 6.12**). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.

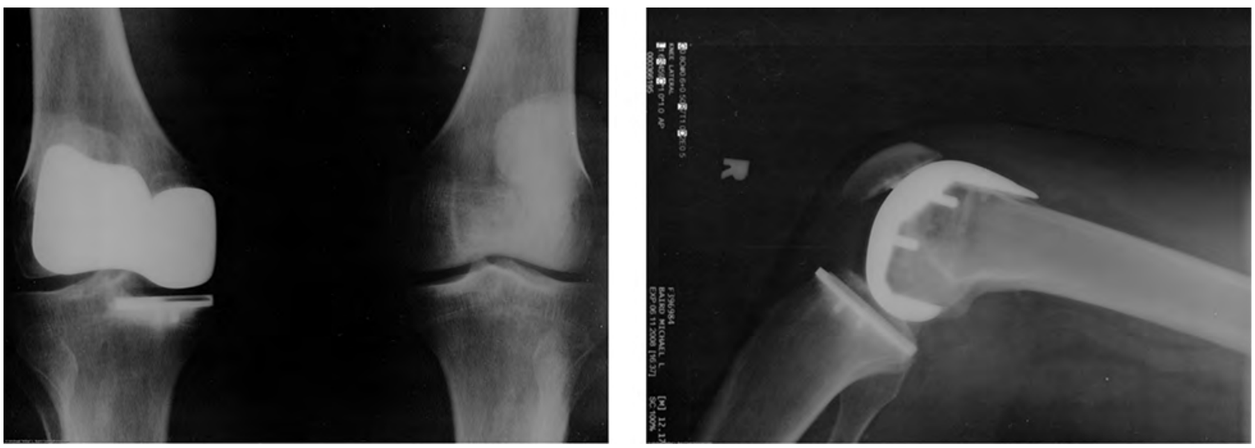


Figure 6.12 Artificial knee replacement is a procedure that has been performed for more than 20 years. These post-operative X-rays show a right knee joint replacement. (credit: Mike Baird)

Natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Hospitals and doctor's clinics commonly use artificial lubricants, such as gels, to reduce friction.

The equations given for static and kinetic friction are empirical laws that describe the behavior of the forces of friction. While these formulas are very useful for practical purposes, they do not have the status of mathematical statements that represent general principles (e.g., Newton's second law). In fact, there are cases for which these equations are not even good approximations. For instance, neither formula is accurate for lubricated surfaces or for two surfaces siding across each other at high speeds. Unless specified, we will not be concerned with these exceptions.

Example 6.10

Static and Kinetic Friction

A 20.0-kg crate is at rest on a floor as shown in **Figure 6.13**. The coefficient of static friction between the crate and floor is 0.700 and the coefficient of kinetic friction is 0.600. A horizontal force $\vec{\mathbf{P}}$ is applied to the crate. Find the force of friction if (a) $\vec{\mathbf{P}} = 20.0\ \text{N}$, (b) $\vec{\mathbf{P}} = 30.0\ \text{N}$, (c) $\vec{\mathbf{P}} = 120.0\ \text{N}$, and (d) $\vec{\mathbf{P}} = 180.0\ \text{N}$.

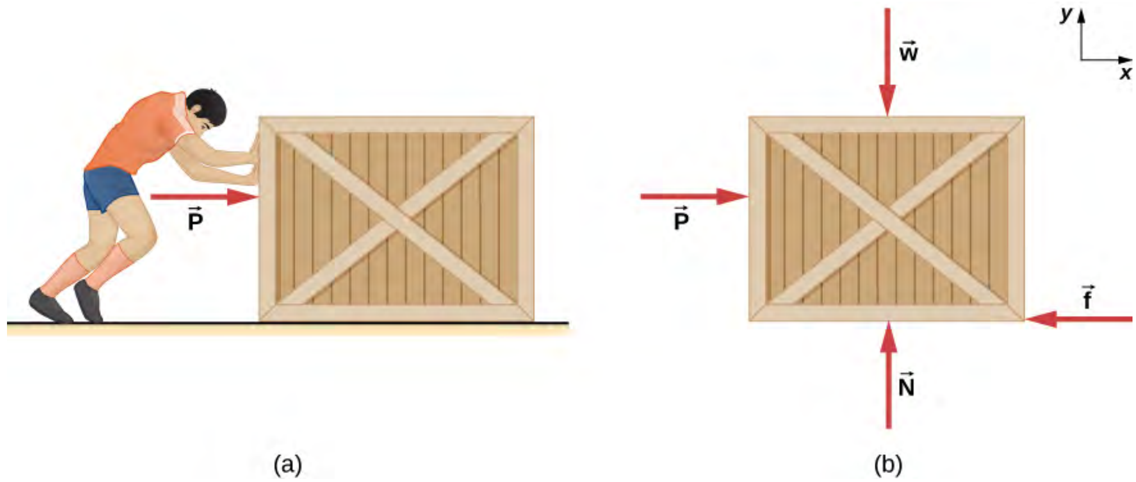


Figure 6.13 (a) A crate on a horizontal surface is pushed with a force $\vec{\mathbf{P}}$. (b) The forces on the crate. Here, $\vec{\mathbf{f}}$ may represent either the static or the kinetic frictional force.

Strategy

The free-body diagram of the crate is shown in **Figure 6.13**(b). We apply Newton's second law in the horizontal and vertical directions, including the friction force in opposition to the direction of motion of the box.

Solution

Newton's second law gives

$$\begin{array}{ll} \sum F_x = ma_x & \sum F_y = ma_y \\ P - f = ma_x & N - w = 0. \end{array}$$

Here we are using the symbol f to represent the frictional force since we have not yet determined whether the crate is subject to station friction or kinetic friction. We do this whenever we are unsure what type of friction is acting. Now the weight of the crate is

$$w = (20.0\,\text{kg})(9.80\,\text{m/s}^2) = 196\,\text{N},$$

which is also equal to N. The maximum force of static friction is therefore $(0.700)(196\,\text{N}) = 137\,\text{N}$. As long as $\vec{\mathbf{P}}$ is less than 137 N, the force of static friction keeps the crate stationary and $f_s = \vec{\mathbf{P}}$. Thus, (a) $f_s = 20.0\,\text{N}$, (b) $f_s = 30.0\,\text{N}$, and (c) $f_s = 120.0\,\text{N}$.

(d) If $\vec{\mathbf{P}} = 180.0\,\text{N}$, the applied force is greater than the maximum force of static friction (137 N), so the crate can no longer remain at rest. Once the crate is in motion, kinetic friction acts. Then

$$f_k = \mu_k N = (0.600)(196\,\text{N}) = 118\,\text{N},$$

and the acceleration is

$$a_x = \frac{\vec{\mathbf{P}} - f_k}{m} = \frac{180.0\,\text{N} - 118\,\text{N}}{20.0\,\text{kg}} = 3.10\,\text{m/s}^2.$$

Significance

This example illustrates how we consider friction in a dynamics problem. Notice that static friction has a value that matches the applied force, until we reach the maximum value of static friction. Also, no motion can occur until the applied force equals the force of static friction, but the force of kinetic friction will then become smaller.

6.7 Check Your Understanding A block of mass 1.0 kg rests on a horizontal surface. The frictional coefficients for the block and surface are $\mu_s = 0.50$ and $\mu_k = 0.40$. (a) What is the minimum horizontal force required to move the block? (b) What is the block's acceleration when this force is applied?

Friction and the Inclined Plane

One situation where friction plays an obvious role is that of an object on a slope. It might be a crate being pushed up a ramp to a loading dock or a skateboarder coasting down a mountain, but the basic physics is the same. We usually generalize the sloping surface and call it an inclined plane but then pretend that the surface is flat. Let's look at an example of analyzing motion on an inclined plane with friction.

Example 6.11

Downhill Skier

A skier with a mass of 62 kg is sliding down a snowy slope at a constant velocity. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

Strategy

The magnitude of kinetic friction is given as 45.0 N. Kinetic friction is related to the normal force N by $f_\text{k} = \mu_\text{k} N$; thus, we can find the coefficient of kinetic friction if we can find the normal force on the skier. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope. (See **Figure 6.14**, which repeats a figure from the chapter on Newton's laws of motion.)

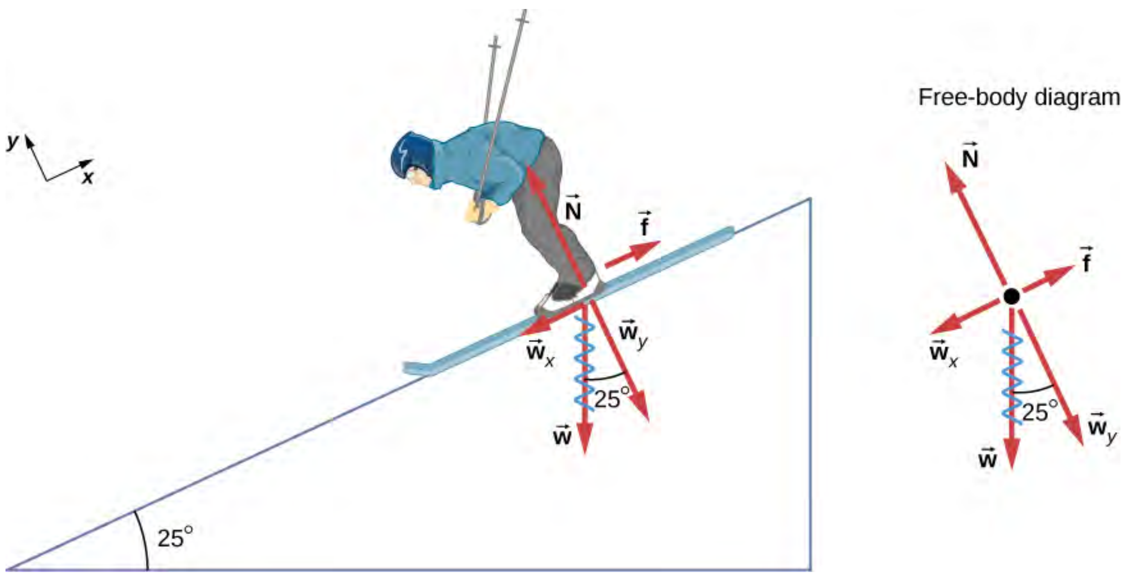


Figure 6.14 The motion of the skier and friction are parallel to the slope, so it is most convenient to project all forces onto a coordinate system where one axis is parallel to the slope and the other is perpendicular (axes shown to left of skier). The normal force $\vec{N}$ is perpendicular to the slope, and friction $\vec{f}$ is parallel to the slope, but the skier's weight $\vec{w}$ has components along both axes, namely $\vec{w}_y$ and $\vec{w}_x$. The normal force $\vec{N}$ is equal in magnitude to $\vec{w}_y$, so there is no motion perpendicular to the slope. However, $\vec{f}$ is less than $\vec{w}_x$ in magnitude, so there is acceleration down the slope (along the *x*-axis).

We have

$$N = w_y = w \cos 25° = mg \cos 25°.$$

Substituting this into our expression for kinetic friction, we obtain

$$f_k = \mu_k mg \cos 25°,$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_k = \frac{f_k}{N} = \frac{f_k}{w \cos 25°} = \frac{f_k}{mg \cos 25°}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_k = \frac{45.0\text{ N}}{(62\text{ kg})(9.80\text{ m/s}^2)(0.906)} = 0.082.$$

Significance

This result is a little smaller than the coefficient listed in **Table 6.1** for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $f_k = \mu_k mg \cos \theta$. All objects slide down a slope with constant acceleration under these circumstances.

We have discussed that when an object rests on a horizontal surface, the normal force supporting it is equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force. When an object is not on a horizontal surface, as with the inclined plane, we must find the force acting on the object that is directed perpendicular to the surface; it is a component of the weight.

We now derive a useful relationship for calculating coefficient of friction on an inclined plane. Notice that the result applies only for situations in which the object slides at constant speed down the ramp.

An object slides down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in **Example 6.10**, the kinetic friction on a slope is $f_k = \mu_k mg \cos \theta$. The component of the weight down the slope is equal to $mg \sin \theta$ (see the free-body diagram in **Figure 6.14**). These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out,

$$\mu_k mg \cos \theta = mg \sin \theta.$$

Solving for μ_k, we find that

$$\mu_k = \frac{mg \sin \theta}{mg \cos \theta} = \tan \theta.$$

Put a coin on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move. Measure the angle of tilt relative to the horizontal and find μ_k. Note that the coin does not start to slide at all until an angle greater than θ is attained, since the coefficient of static friction is larger than the coefficient of kinetic friction. Think about how this may affect the value for μ_k and its uncertainty.

Atomic-Scale Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds of billions of dollars in energy which is currently being converted (unnecessarily) into heat.

Figure 6.15 illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the amount of area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area because only high spots touch. When a greater normal force is exerted, the actual contact area increases, and we find that the friction is proportional to this area.

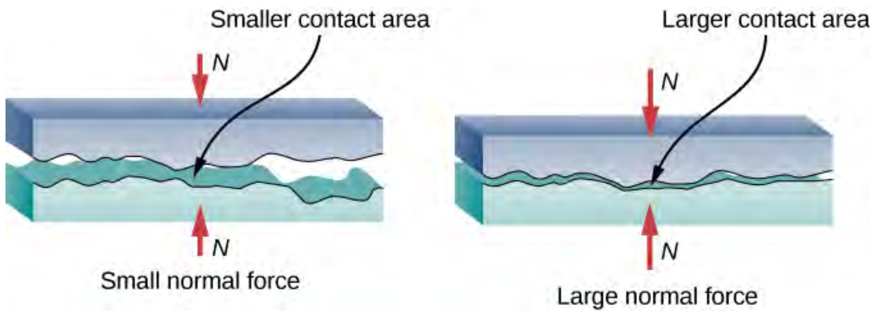


Figure 6.15 Two rough surfaces in contact have a much smaller area of actual contact than their total area. When the normal force is larger as a result of a larger applied force, the area of actual contact increases, as does friction.

However, the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed? Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance, and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. **Figure 6.16** shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which is discussed in **Static Equilibrium and Elasticity**. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.

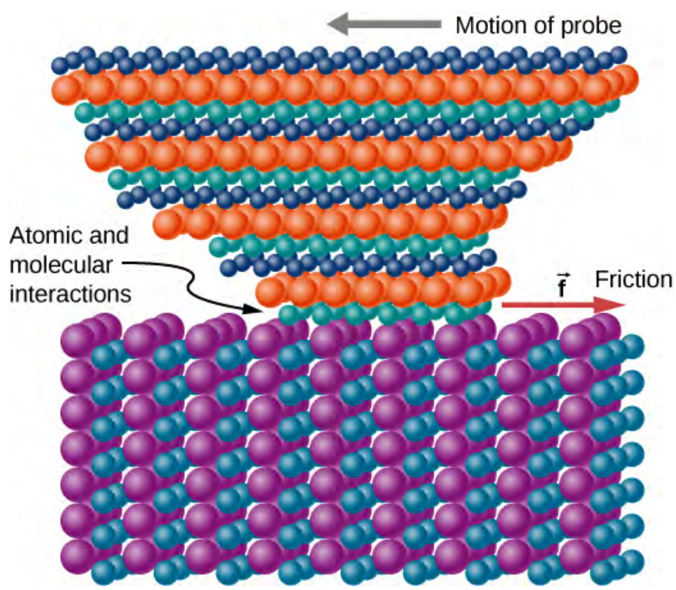


Figure 6.16 The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

Describe a **model for friction (https://openstaxcollege.org/l/21friction)** on a molecular level. Describe matter in terms of molecular motion. The description should include diagrams to support the description; how the temperature affects the image; what are the differences and similarities between solid, liquid, and gas particle motion; and how the size and speed of gas molecules relate to everyday objects.

Example 6.12

Sliding Blocks

The two blocks of **Figure 6.17** are attached to each other by a massless string that is wrapped around a frictionless pulley. When the bottom 4.00-kg block is pulled to the left by the constant force $\vec{\mathbf{P}}$, the top 2.00-kg block slides across it to the right. Find the magnitude of the force necessary to move the blocks at constant speed. Assume that the coefficient of kinetic friction between all surfaces is 0.400.

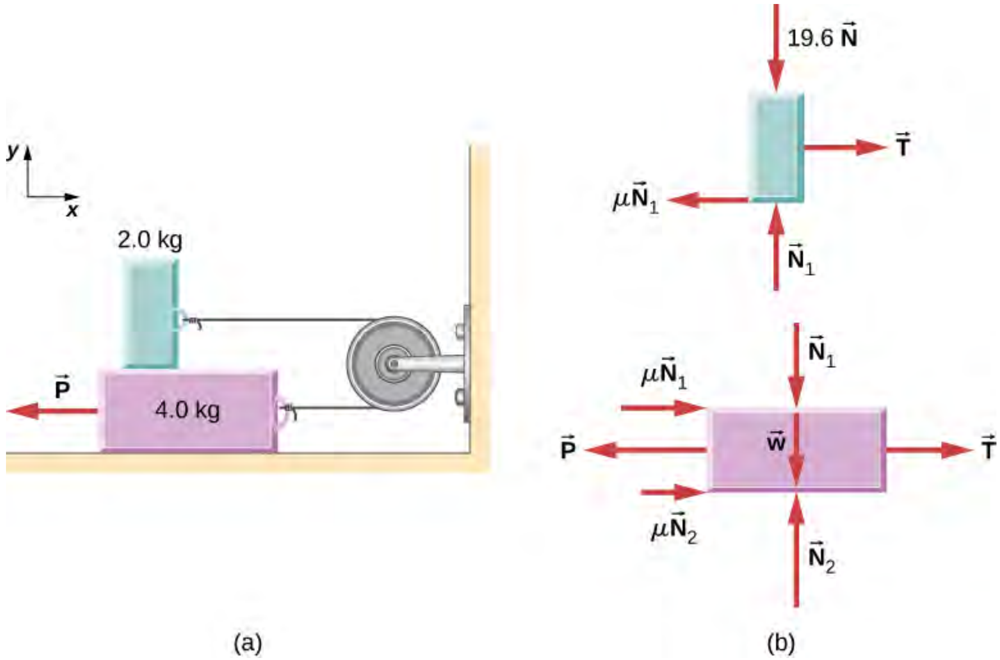


Figure 6.17 (a) Each block moves at constant velocity. (b) Free-body diagrams for the blocks.

Strategy

We analyze the motions of the two blocks separately. The top block is subjected to a contact force exerted by the bottom block. The components of this force are the normal force N_1 and the frictional force $-0.400N_1$. Other forces on the top block are the tension $T\text{i}$ in the string and the weight of the top block itself, 19.6 N. The bottom block is subjected to contact forces due to the top block and due to the floor. The first contact force has components $-N_1$ and $0.400N_1$, which are simply reaction forces to the contact forces that the bottom block exerts on the top block. The components of the contact force of the floor are N_2 and $0.400N_2$. Other forces on this block are $-P$, the tension $T\text{i}$, and the weight –39.2 N.

Solution

Since the top block is moving horizontally to the right at constant velocity, its acceleration is zero in both the horizontal and the vertical directions. From Newton's second law,

$$\begin{aligned} \sum F_x &= m_1 a_x & \qquad \sum F_y &= m_1 a_y \\ T - 0.400N_1 &= 0 & N_1 - 19.6\,\text{N} &= 0. \end{aligned}$$

Solving for the two unknowns, we obtain $N_1 = 19.6\,\text{N}$ and $T = 0.40N_1 = 7.84\,\text{N}$. The bottom block is also not accelerating, so the application of Newton's second law to this block gives

$$\begin{aligned} &\sum F_x = m_2 a_x & &\sum F_y = m_2 a_y \\ &T - P + 0.400\,N_1 + 0.400\,N_2 = 0 & &N_2 - 39.2\,\text{N} - N_1 = 0. \end{aligned}$$

The values of N_1 and T were found with the first set of equations. When these values are substituted into the second set of equations, we can determine N_2 and P. They are

$$N_2 = 58.8\text{ N} \text{ and } P = 39.2\text{ N}.$$

Significance

Understanding what direction in which to draw the friction force is often troublesome. Notice that each friction force labeled in **Figure 6.17** acts in the direction opposite the motion of its corresponding block.

Example 6.13

A Crate on an Accelerating Truck

A 50.0-kg crate rests on the bed of a truck as shown in **Figure 6.18**. The coefficients of friction between the surfaces are $\mu_k = 0.300$ and $\mu_s = 0.400$. Find the frictional force on the crate when the truck is accelerating forward relative to the ground at (a) 2.00 m/s^2, and (b) 5.00 m/s^2.

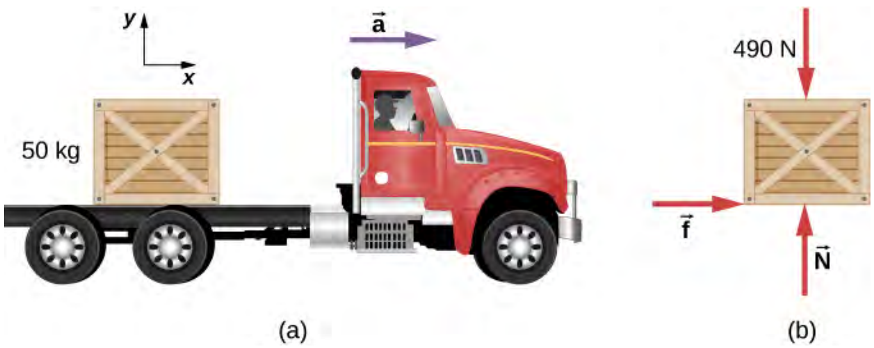


Figure 6.18 (a) A crate rests on the bed of the truck that is accelerating forward. (b) The free-body diagram of the crate.

Strategy

The forces on the crate are its weight and the normal and frictional forces due to contact with the truck bed. We start by *assuming* that the crate is not slipping. In this case, the static frictional force f_s acts on the crate. Furthermore, the accelerations of the crate and the truck are equal.

Solution

a. Application of Newton's second law to the crate, using the reference frame attached to the ground, yields

$$\begin{aligned} \sum F_x &= ma_x \\ f_s &= (50.0\text{ kg})(2.00\text{ m/s}^2) \\ &= 1.00 \times 10^2\text{ N} \end{aligned} \qquad \begin{aligned} \sum F_y &= ma_y \\ N - 4.90 \times 10^2\text{ N} &= (50.0\text{ kg})(0) \\ N &= 4.90 \times 10^2\text{ N}. \end{aligned}$$

We can now check the validity of our no-slip assumption. The maximum value of the force of static friction is

$$\mu_s N = (0.400)(4.90 \times 10^2\text{ N}) = 196\text{ N},$$

whereas the *actual* force of static friction that acts when the truck accelerates forward at 2.00 m/s^2 is only $1.00 \times 10^2\text{ N}$. Thus, the assumption of no slipping is valid.

b. If the crate is to move with the truck when it accelerates at 5.0 m/s^2, the force of static friction must be

$$f_s = ma_x = (50.0\,\text{kg})(5.00\,\text{m/s}^2) = 250\,\text{N}.$$

Since this exceeds the maximum of 196 N, the crate must slip. The frictional force is therefore kinetic and is

$$f_k = \mu_k N = (0.300)(4.90 \times 10^2\ \text{N}) = 147\,\text{N}.$$

The horizontal acceleration of the crate relative to the ground is now found from

$$\begin{aligned} \sum F_x &= ma_x \\ 147\,\text{N} &= (50.0\,\text{kg})a_x, \\ \text{so } a_x &= 2.94\,\text{m/s}^2. \end{aligned}$$

Significance

Relative to the ground, the truck is accelerating forward at $5.0\,\text{m/s}^2$ and the crate is accelerating forward at $2.94\,\text{m/s}^2$. Hence the crate is sliding backward relative to the bed of the truck with an acceleration $2.94\,\text{m/s}^2 - 5.00\,\text{m/s}^2 = -2.06\,\text{m/s}^2$.

Example 6.14

Snowboarding

Earlier, we analyzed the situation of a downhill skier moving at constant velocity to determine the coefficient of kinetic friction. Now let's do a similar analysis to determine acceleration. The snowboarder of **Figure 6.19** glides down a slope that is inclined at $\theta = 13^0$ to the horizontal. The coefficient of kinetic friction between the board and the snow is $\mu_\text{k} = 0.20$. What is the acceleration of the snowboarder?

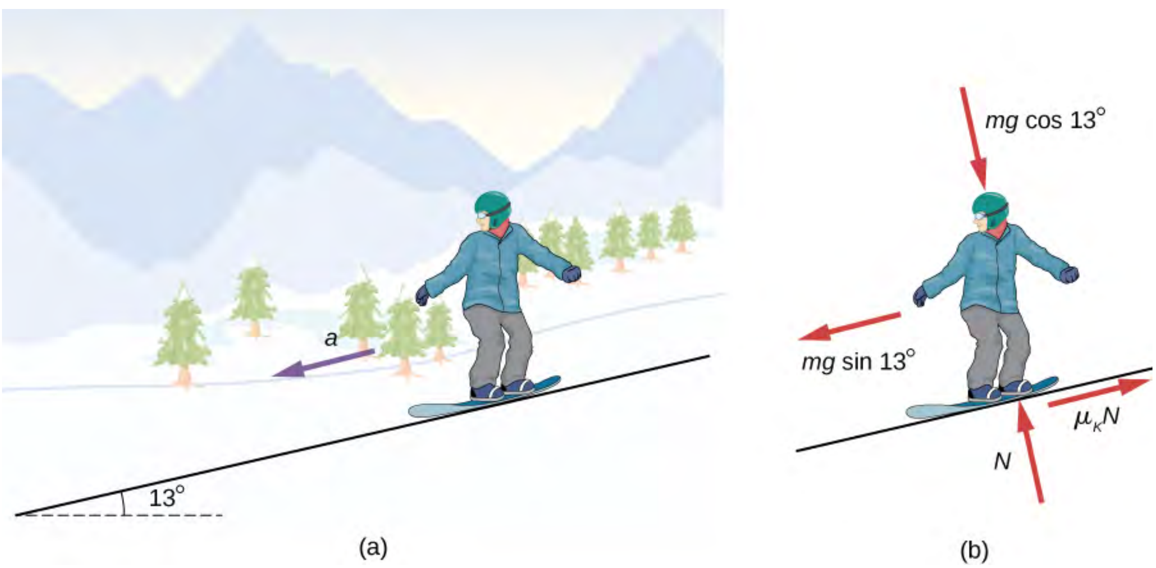


Figure 6.19 (a) A snowboarder glides down a slope inclined at 13° to the horizontal. (b) The free-body diagram of the snowboarder.

Strategy

The forces acting on the snowboarder are her weight and the contact force of the slope, which has a component normal to the incline and a component along the incline (force of kinetic friction). Because she moves along the slope, the most convenient reference frame for analyzing her motion is one with the *x*-axis along and the *y*-axis perpendicular to the incline. In this frame, both the normal and the frictional forces lie along coordinate axes, the components of the weight are $mg \sin \theta$ along the slope and $mg \cos \theta$ at right angles into the slope, and the only acceleration is along the *x*-axis $(a_y = 0)$.

Solution

We can now apply Newton's second law to the snowboarder:

$$\begin{aligned} \sum F_x &= ma_x \qquad \sum F_y = ma_y \\ mg \sin \theta - \mu_k N &= ma_x \qquad N - mg \cos \theta = m(0). \end{aligned}$$

From the second equation, $N = mg \cos \theta$. Upon substituting this into the first equation, we find

$$\begin{aligned} a_x &= g(\sin \theta - \mu_{\text{k}} \cos \theta) \\ &= g(\sin 13° - 0.20 \cos 13°) = 0.29 \text{ m/s}^2. \end{aligned}$$

Significance

Notice from this equation that if θ is small enough or μ_{k} is large enough, a_x is negative, that is, the snowboarder slows down.

6.8 Check Your Understanding The snowboarder is now moving down a hill with incline $10.0°$. What is the skier's acceleration?

6.3 | Centripetal Force

Learning Objectives

By the end of the section, you will be able to:

- Explain the equation for centripetal acceleration
- Apply Newton's second law to develop the equation for centripetal force
- Use circular motion concepts in solving problems involving Newton's laws of motion

In **Motion in Two and Three Dimensions**, we examined the basic concepts of circular motion. An object undergoing circular motion, like one of the race cars shown at the beginning of this chapter, must be accelerating because it is changing the direction of its velocity. We proved that this centrally directed acceleration, called centripetal acceleration, is given by the formula

$$a_{\text{c}} = \frac{v^2}{r}$$

where *v* is the velocity of the object, directed along a tangent line to the curve at any instant. If we know the angular velocity ω, then we can use

$$a_{\text{c}} = r\omega^2.$$

Angular velocity gives the rate at which the object is turning through the curve, in units of rad/s. This acceleration acts along the radius of the curved path and is thus also referred to as a radial acceleration.

An acceleration must be produced by a force. Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, friction between roller skates and a rink floor, a banked roadway's force on a car, and forces on the tube of a spinning centrifuge. Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton's second law of motion, net force is mass times acceleration: $F_{\text{net}} = ma$. For uniform circular motion, the acceleration is the centripetal acceleration: $a = a_c$. Thus, the magnitude of centripetal force F_c is

$$F_c = ma_c.$$

By substituting the expressions for centripetal acceleration a_c $(a_c = \frac{v^2}{r};\ a_c = r\omega^2)$, we get two expressions for the centripetal force F_c in terms of mass, velocity, angular velocity, and radius of curvature:

$$F_c = m\frac{v^2}{r};\quad F_c = mr\omega^2. \tag{6.3}$$

You may use whichever expression for centripetal force is more convenient. Centripetal force $\vec{\mathbf{F}}_c$ is always perpendicular to the path and points to the center of curvature, because $\vec{\mathbf{a}}_c$ is perpendicular to the velocity and points to the center of curvature. Note that if you solve the first expression for *r*, you get

$$r = \frac{mv^2}{F_c}.$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve, as in **Figure 6.20**.

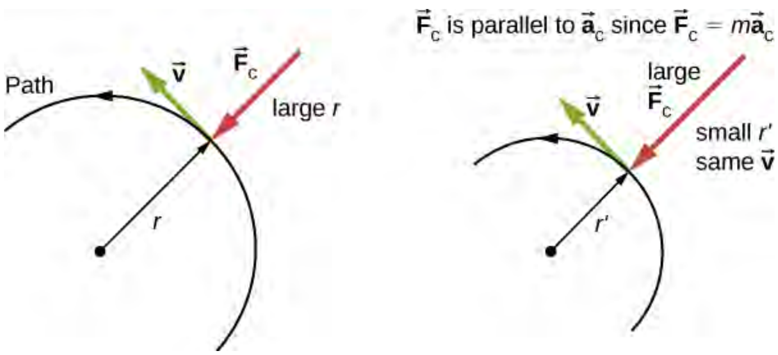


Figure 6.20 The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the F_c, the smaller the radius of curvature *r* and the sharper the curve. The second curve has the same *v*, but a larger F_c produces a smaller *r*′.

Example 6.15

What Coefficient of Friction Do Cars Need on a Flat Curve?

(a) Calculate the centripetal force exerted on a 900.0-kg car that negotiates a 500.0-m radius curve at 25.00 m/s.
(b) Assuming an unbanked curve, find the minimum static coefficient of friction between the tires and the road, static friction being the reason that keeps the car from slipping (**Figure 6.21**).

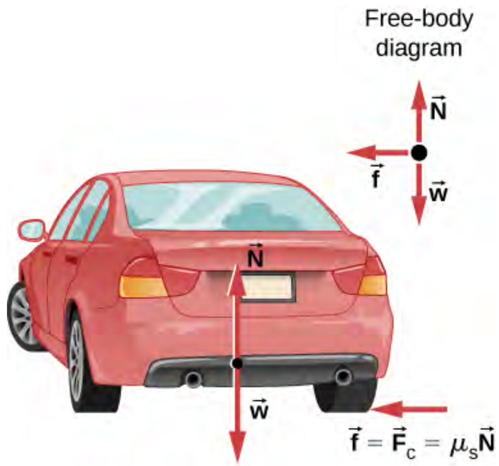


Figure 6.21 This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

Strategy

a. We know that $F_c = \frac{mv^2}{r}$. Thus,

$$F_c = \frac{mv^2}{r} = \frac{(900.0\ \text{kg})(25.00\ \text{m/s})^2}{(500.0\ \text{m})} = 1125\ \text{N}.$$

b. **Figure 6.21** shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_s N$, where μ_s is the static coefficient of friction and *N* is the normal force. The normal force equals the car's weight on level ground, so $N = mg$. Thus the centripetal force in this situation is

$$F_c = f = \mu_s N = \mu_s mg.$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the equation

$$F_c = m\frac{v^2}{r},$$

we obtain

$$m\frac{v^2}{r} = \mu_s mg.$$

We solve this for μ_s, noting that mass cancels, and obtain

$$\mu_s = \frac{v^2}{rg}.$$

Substituting the knowns,

$$\mu_s = \frac{(25.00\ \text{m/s})^2}{(500.0\ \text{m})(9.80\ \text{m/s}^2)} = 0.13.$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Significance

The coefficient of friction found in **Figure 6.21**(b) is much smaller than is typically found between tires and roads. The car still negotiates the curve if the coefficient is greater than 0.13, because static friction is a responsive force, able to assume a value less than but no more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that, in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less, as discussed next.

6.9 Check Your Understanding A car moving at 96.8 km/h travels around a circular curve of radius 182.9 m on a flat country road. What must be the minimum coefficient of static friction to keep the car from slipping?

Banked Curves

Let us now consider **banked curves**, where the slope of the road helps you negotiate the curve (**Figure 6.22**). The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an "ideally banked curve," the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. We will derive an expression for θ for an ideally banked curve and consider an example related to it.

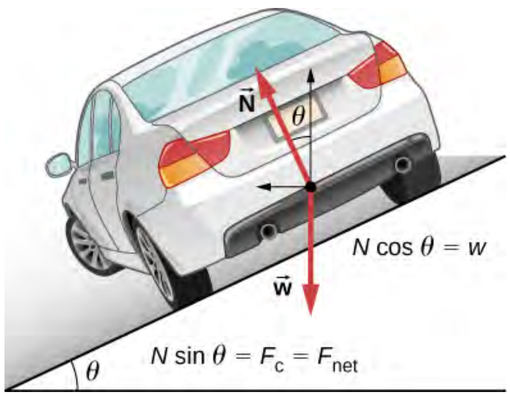


Figure 6.22 The car on this banked curve is moving away and turning to the left.

For **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force N in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively. In cases in which forces are not parallel, it is most convenient to consider components along perpendicular axes—in this case, the vertical and horizontal directions.

Figure 6.22 shows a free-body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force equals the necessary centripetal force. The only two external forces acting on the car are its weight $\vec{\mathbf{w}}$ and the normal force of the road $\vec{\mathbf{N}}$. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. Because this is the crucial force and it is horizontal, we use a coordinate system with vertical and horizontal axes. Only the normal force has a horizontal component, so this must equal the centripetal force, that is,

$$N \sin\theta = \frac{mv^2}{r}.$$

Because the car does not leave the surface of the road, the net vertical force must be zero, meaning that the vertical components of the two external forces must be equal in magnitude and opposite in direction. From **Figure 6.22**, we see that the vertical component of the normal force is $N\cos\theta$, and the only other vertical force is the car's weight. These must be equal in magnitude; thus,

$$N\cos\theta = mg.$$

Now we can combine these two equations to eliminate N and get an expression for θ, as desired. Solving the second equation for $N = mg/(cos\theta)$ and substituting this into the first yields

$$\begin{aligned} mg\frac{\sin\theta}{\cos\theta} &= \frac{mv^2}{r} \\ mg\tan\theta &= \frac{mv^2}{r} \\ \tan\theta &= \frac{v^2}{rg}. \end{aligned}$$

Taking the inverse tangent gives

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right). \qquad (6.4)$$

This expression can be understood by considering how θ depends on v and r. A large θ is obtained for a large v and a small r. That is, roads must be steeply banked for high speeds and sharp curves. Friction helps, because it allows you to take the curve at greater or lower speed than if the curve were frictionless. Note that θ does not depend on the mass of the vehicle.

Example 6.16

What Is the Ideal Speed to Take a Steeply Banked Tight Curve?

Curves on some test tracks and race courses, such as Daytona International Speedway in Florida, are very steeply banked. This banking, with the aid of tire friction and very stable car configurations, allows the curves to be taken at very high speed. To illustrate, calculate the speed at which a 100.0-m radius curve banked at $31.0°$ should be driven if the road were frictionless.

Strategy

We first note that all terms in the expression for the ideal angle of a banked curve except for speed are known; thus, we need only rearrange it so that speed appears on the left-hand side and then substitute known quantities.

Solution

Starting with

$$\tan\theta = \frac{v^2}{rg},$$

we get

$$v = \sqrt{rg\tan\theta}.$$

Noting that $\tan 31.0° = 0.609$, we obtain

$$v = \sqrt{(100.0\text{ m})(9.80\text{ m/s}^2)(0.609)} = 24.4\text{ m/s}.$$

Significance

This is just about 165 km/h, consistent with a very steeply banked and rather sharp curve. Tire friction enables a vehicle to take the curve at significantly higher speeds.

Airplanes also make turns by banking. The lift force, due to the force of the air on the wing, acts at right angles to the wing. When the airplane banks, the pilot is obtaining greater lift than necessary for level flight. The vertical component of lift balances the airplane's weight, and the horizontal component accelerates the plane. The banking angle shown in **Figure 6.23** is given by θ. We analyze the forces in the same way we treat the case of the car rounding a banked curve.

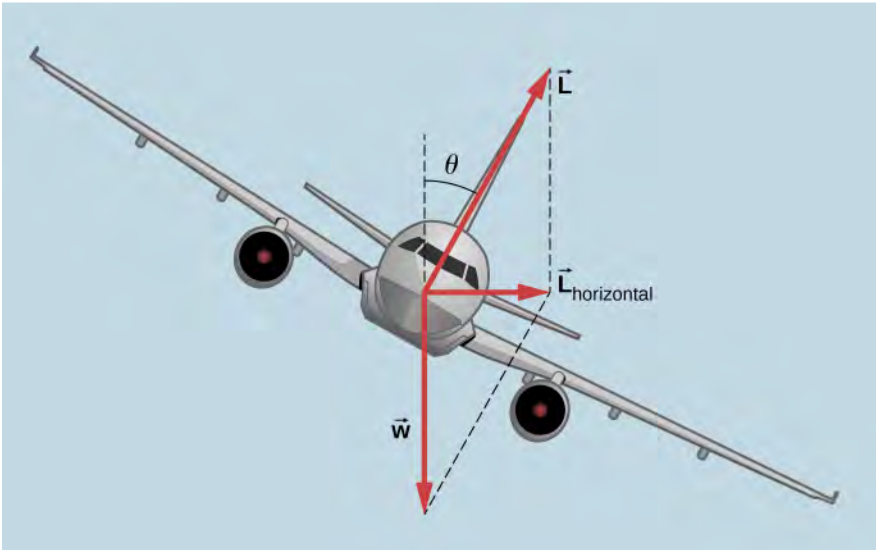


Figure 6.23 In a banked turn, the horizontal component of lift is unbalanced and accelerates the plane. The normal component of lift balances the plane's weight. The banking angle is given by θ. Compare the vector diagram with that shown in **Figure 6.22**.

Join the **ladybug (https://openstaxcollege.org/l/21ladybug)** in an exploration of rotational motion. Rotate the merry-go-round to change its angle or choose a constant angular velocity or angular acceleration. Explore how circular motion relates to the bug's *xy*-position, velocity, and acceleration using vectors or graphs.

A circular motion requires a force, the so-called centripetal force, which is directed to the axis of rotation. This simplified **model of a carousel (https://openstaxcollege.org/l/21carousel)** demonstrates this force.

Inertial Forces and Noninertial (Accelerated) Frames: The Coriolis Force

What do taking off in a jet airplane, turning a corner in a car, riding a merry-go-round, and the circular motion of a tropical cyclone have in common? Each exhibits inertial forces—forces that merely seem to arise from motion, because the observer's frame of reference is accelerating or rotating. When taking off in a jet, most people would agree it feels as if you are being pushed back into the seat as the airplane accelerates down the runway. Yet a physicist would say that *you* tend to remain stationary while the *seat* pushes forward on you. An even more common experience occurs when you make a tight curve in your car—say, to the right (**Figure 6.24**). You feel as if you are thrown (that is, *forced*) toward the left relative to the car. Again, a physicist would say that *you* are going in a straight line (recall Newton's first law) but the *car* moves to the right, not that you are experiencing a force from the left.

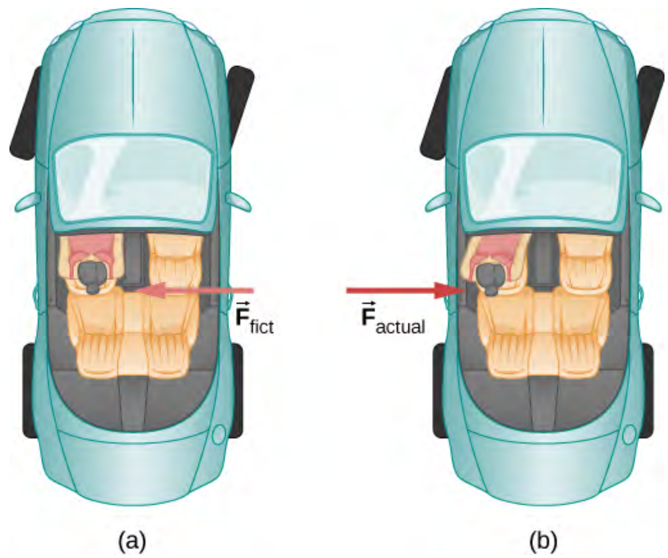


Figure 6.24 (a) The car driver feels herself forced to the left relative to the car when she makes a right turn. This is an inertial force arising from the use of the car as a frame of reference. (b) In Earth's frame of reference, the driver moves in a straight line, obeying Newton's first law, and the car moves to the right. There is no force to the left on the driver relative to Earth. Instead, there is a force to the right on the car to make it turn.

We can reconcile these points of view by examining the frames of reference used. Let us concentrate on people in a car. Passengers instinctively use the car as a frame of reference, whereas a physicist might use Earth. The physicist might make this choice because Earth is nearly an inertial frame of reference, in which all forces have an identifiable physical origin. In such a frame of reference, Newton's laws of motion take the form given in Newton's Laws of Motion. The car is a **noninertial frame of reference** because it is accelerated to the side. The force to the left sensed by car passengers is an **inertial force** having no physical origin (it is due purely to the inertia of the passenger, not to some physical cause such as tension, friction, or gravitation). The car, as well as the driver, is actually accelerating to the right. This inertial force is said to be an inertial force because it does not have a physical origin, such as gravity.

A physicist will choose whatever reference frame is most convenient for the situation being analyzed. There is no problem to a physicist in including inertial forces and Newton's second law, as usual, if that is more convenient, for example, on a merry-go-round or on a rotating planet. Noninertial (accelerated) frames of reference are used when it is useful to do so. Different frames of reference must be considered in discussing the motion of an astronaut in a spacecraft traveling at speeds near the speed of light, as you will appreciate in the study of the special theory of relativity.

Let us now take a mental ride on a merry-go-round—specifically, a rapidly rotating playground merry-go-round (Figure 6.25). You take the merry-go-round to be your frame of reference because you rotate together. When rotating in that noninertial frame of reference, you feel an inertial force that tends to throw you off; this is often referred to as a *centrifugal force* (not to be confused with centripetal force). Centrifugal force is a commonly used term, but it does not actually exist. You must hang on tightly to counteract your inertia (which people often refer to as centrifugal force). In Earth's frame of reference, there is no force trying to throw you off; we emphasize that centrifugal force is a fiction. You must hang on to make yourself go in a circle because otherwise you would go in a straight line, right off the merry-go-round, in keeping with Newton's first law. But the force you exert acts toward the center of the circle.

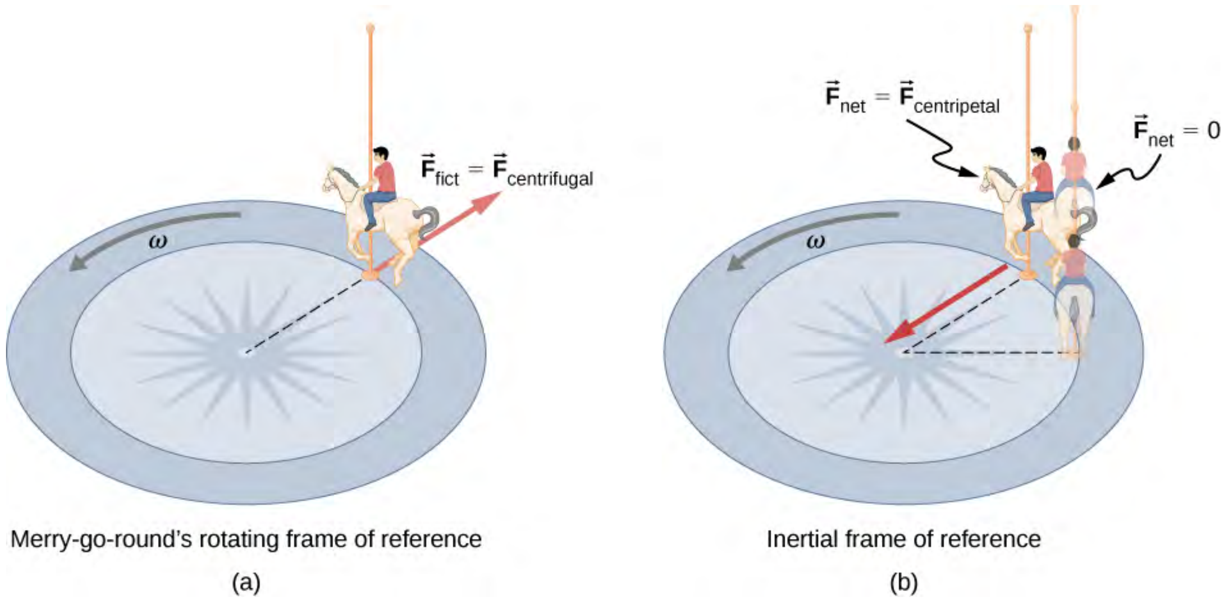


Figure 6.25 (a) A rider on a merry-go-round feels as if he is being thrown off. This inertial force is sometimes mistakenly called the centrifugal force in an effort to explain the rider's motion in the rotating frame of reference. (b) In an inertial frame of reference and according to Newton's laws, it is his inertia that carries him off (the unshaded rider has $F_{\text{net}} = 0$ and heads in a straight line). A force, $F_{\text{centripetal}}$, is needed to cause a circular path.

This inertial effect, carrying you away from the center of rotation if there is no centripetal force to cause circular motion, is put to good use in centrifuges (**Figure 6.26**). A centrifuge spins a sample very rapidly, as mentioned earlier in this chapter. Viewed from the rotating frame of reference, the inertial force throws particles outward, hastening their sedimentation. The greater the angular velocity, the greater the centrifugal force. But what really happens is that the inertia of the particles carries them along a line tangent to the circle while the test tube is forced in a circular path by a centripetal force.

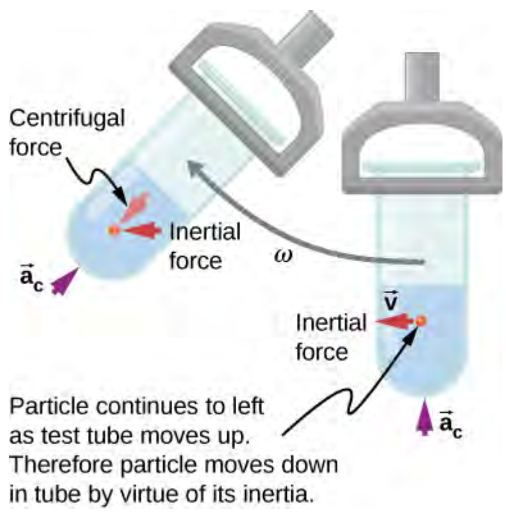


Figure 6.26 Centrifuges use inertia to perform their task. Particles in the fluid sediment settle out because their inertia carries them away from the center of rotation. The large angular velocity of the centrifuge quickens the sedimentation. Ultimately, the particles come into contact with the test tube walls, which then supply the centripetal force needed to make them move in a circle of constant radius.

Let us now consider what happens if something moves in a rotating frame of reference. For example, what if you slide a ball directly away from the center of the merry-go-round, as shown in **Figure 6.27**? The ball follows a straight path relative to

Earth (assuming negligible friction) and a path curved to the right on the merry-go-round's surface. A person standing next to the merry-go-round sees the ball moving straight and the merry-go-round rotating underneath it. In the merry-go-round's frame of reference, we explain the apparent curve to the right by using an inertial force, called the **Coriolis force**, which causes the ball to curve to the right. The Coriolis force can be used by anyone in that frame of reference to explain why objects follow curved paths and allows us to apply Newton's laws in noninertial frames of reference.

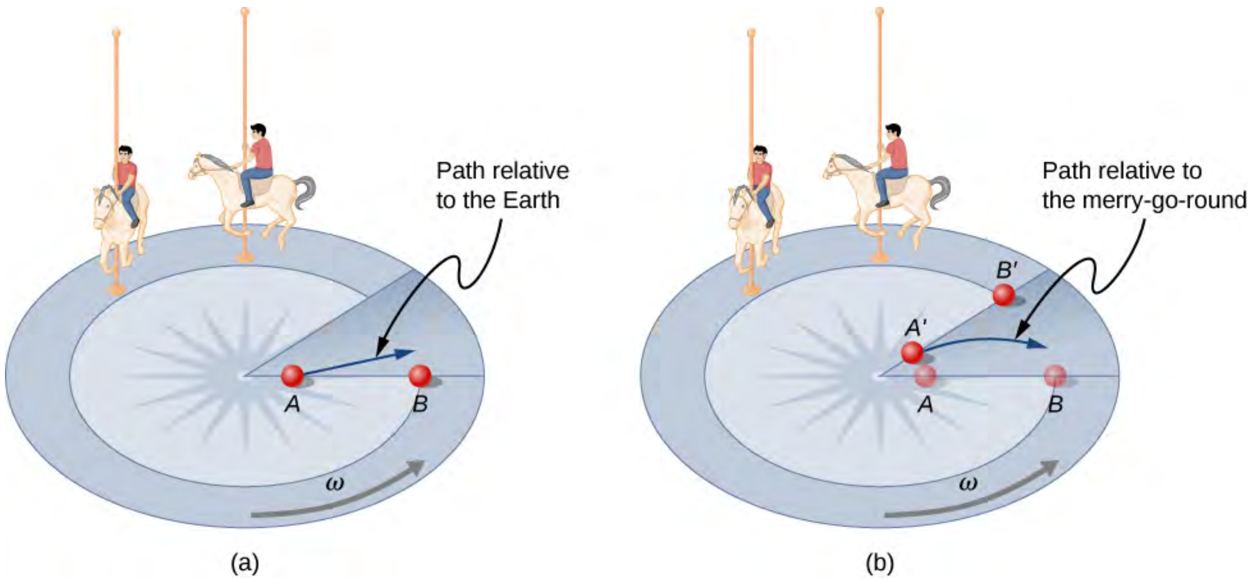


Figure 6.27 Looking down on the counterclockwise rotation of a merry-go-round, we see that a ball slid straight toward the edge follows a path curved to the right. The person slides the ball toward point *B*, starting at point *A*. Both points rotate to the shaded positions (*A*' and *B*') shown in the time that the ball follows the curved path in the rotating frame and a straight path in Earth's frame.

Up until now, we have considered Earth to be an inertial frame of reference with little or no worry about effects due to its rotation. Yet such effects *do* exist—in the rotation of weather systems, for example. Most consequences of Earth's rotation can be qualitatively understood by analogy with the merry-go-round. Viewed from above the North Pole, Earth rotates counterclockwise, as does the merry-go-round in **Figure 6.27**. As on the merry-go-round, any motion in Earth's Northern Hemisphere experiences a Coriolis force to the right. Just the opposite occurs in the Southern Hemisphere; there, the force is to the left. Because Earth's angular velocity is small, the Coriolis force is usually negligible, but for large-scale motions, such as wind patterns, it has substantial effects.

The Coriolis force causes hurricanes in the Northern Hemisphere to rotate in the counterclockwise direction, whereas tropical cyclones in the Southern Hemisphere rotate in the clockwise direction. (The terms hurricane, typhoon, and tropical storm are regionally specific names for cyclones, which are storm systems characterized by low pressure centers, strong winds, and heavy rains.) **Figure 6.28** helps show how these rotations take place. Air flows toward any region of low pressure, and tropical cyclones contain particularly low pressures. Thus winds flow toward the center of a tropical cyclone or a low-pressure weather system at the surface. In the Northern Hemisphere, these inward winds are deflected to the right, as shown in the figure, producing a counterclockwise circulation at the surface for low-pressure zones of any type. Low pressure at the surface is associated with rising air, which also produces cooling and cloud formation, making low-pressure patterns quite visible from space. Conversely, wind circulation around high-pressure zones is clockwise in the Southern Hemisphere but is less visible because high pressure is associated with sinking air, producing clear skies.

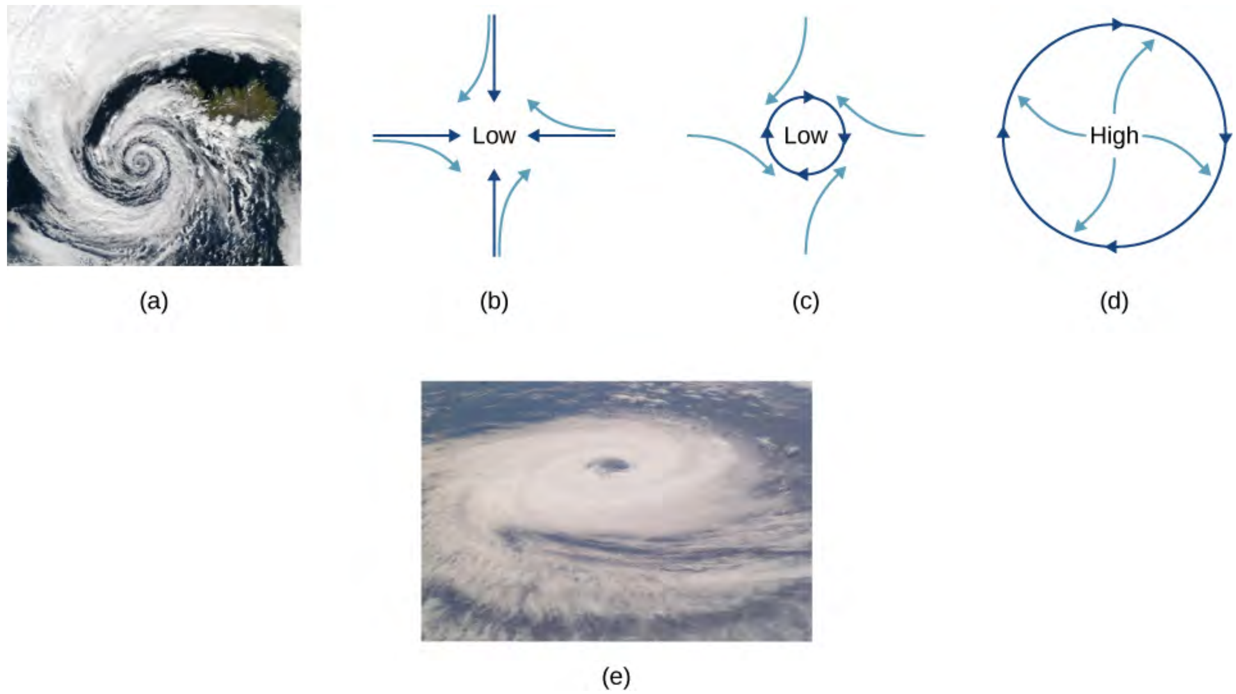


Figure 6.28 (a) The counterclockwise rotation of this Northern Hemisphere hurricane is a major consequence of the Coriolis force. (b) Without the Coriolis force, air would flow straight into a low-pressure zone, such as that found in tropical cyclones. (c) The Coriolis force deflects the winds to the right, producing a counterclockwise rotation. (d) Wind flowing away from a high-pressure zone is also deflected to the right, producing a clockwise rotation. (e) The opposite direction of rotation is produced by the Coriolis force in the Southern Hemisphere, leading to tropical cyclones. (credit a and credit e: modifications of work by NASA)

The rotation of tropical cyclones and the path of a ball on a merry-go-round can just as well be explained by inertia and the rotation of the system underneath. When noninertial frames are used, inertial forces, such as the Coriolis force, must be invented to explain the curved path. There is no identifiable physical source for these inertial forces. In an inertial frame, inertia explains the path, and no force is found to be without an identifiable source. Either view allows us to describe nature, but a view in an inertial frame is the simplest in the sense that all forces have origins and explanations.

6.4 | Drag Force and Terminal Speed

Learning Objectives

By the end of the section, you will be able to:

- Express the drag force mathematically
- Describe applications of the drag force
- Define terminal velocity
- Determine an object’s terminal velocity given its mass

Another interesting force in everyday life is the force of drag on an object when it is moving in a fluid (either a gas or a liquid). You feel the drag force when you move your hand through water. You might also feel it if you move your hand during a strong wind. The faster you move your hand, the harder it is to move. You feel a smaller drag force when you tilt your hand so only the side goes through the air—you have decreased the area of your hand that faces the direction of motion.

Drag Forces

Like friction, the **drag force** always opposes the motion of an object. Unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid. This functionality is complicated and depends upon the shape of the object, its size, its velocity, and the fluid it is in. For most large objects such as cyclists, cars, and baseballs not moving too slowly, the magnitude of the drag force F_D is proportional to the square of the speed of the object. We can write this relationship mathematically as $F_D \propto v^2$. When taking into account other factors, this relationship becomes

$$F_D = \frac{1}{2}C\rho A v^2, \tag{6.5}$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid. (Recall that density is mass per unit volume.) This equation can also be written in a more generalized fashion as $F_D = bv^2$, where b is a constant equivalent to $0.5C\rho A$. We have set the exponent n for these equations as 2 because when an object is moving at high velocity through air, the magnitude of the drag force is proportional to the square of the speed. As we shall see in **Fluid Mechanics**, for small particles moving at low speeds in a fluid, the exponent n is equal to 1.

Drag Force

Drag force F_D is proportional to the square of the speed of the object. Mathematically,

$$F_D = \frac{1}{2}C\,\rho\,Av^2,$$

where C is the drag coefficient, A is the area of the object facing the fluid, and ρ is the density of the fluid.

Athletes as well as car designers seek to reduce the drag force to lower their race times (**Figure 6.29**). Aerodynamic shaping of an automobile can reduce the drag force and thus increase a car’s gas mileage.

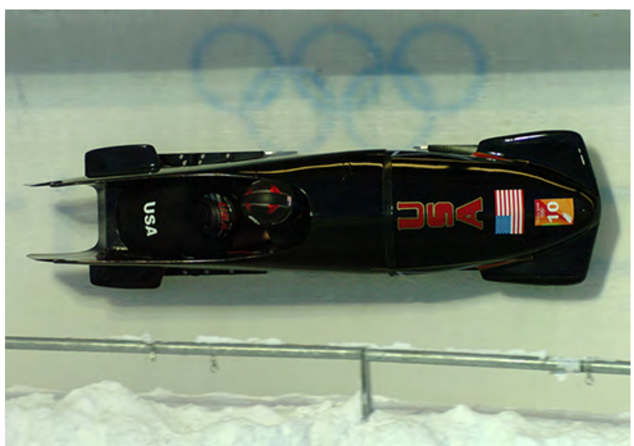


Figure 6.29 From racing cars to bobsled racers, aerodynamic shaping is crucial to achieving top speeds. Bobsleds are designed for speed and are shaped like a bullet with tapered fins. (credit: “U.S. Army”/Wikimedia Commons)

The value of the drag coefficient C is determined empirically, usually with the use of a wind tunnel (**Figure 6.30**).

Figure 6.30 NASA researchers test a model plane in a wind tunnel. (credit: NASA/Ames)

The drag coefficient can depend upon velocity, but we assume that it is a constant here. **Table 6.2** lists some typical drag coefficients for a variety of objects. Notice that the drag coefficient is a dimensionless quantity. At highway speeds, over 50% of the power of a car is used to overcome air drag. The most fuel-efficient cruising speed is about 70–80 km/h (about 45–50 mi/h). For this reason, during the 1970s oil crisis in the United States, maximum speeds on highways were set at about 90 km/h (55 mi/h).

Object	*C*
Airfoil	0.05
Toyota Camry	0.28
Ford Focus	0.32
Honda Civic	0.36
Ferrari Testarossa	0.37
Dodge Ram Pickup	0.43
Sphere	0.45
Hummer H2 SUV	0.64
Skydiver (feet first)	0.70
Bicycle	0.90
Skydiver (horizontal)	1.0
Circular flat plate	1.12

Table 6.2 Typical Values of Drag Coefficient *C*

Substantial research is under way in the sporting world to minimize drag. The dimples on golf balls are being redesigned, as are the clothes that athletes wear. Bicycle racers and some swimmers and runners wear full bodysuits. Australian Cathy Freeman wore a full body suit in the 2000 Sydney Olympics and won a gold medal in the 400-m race. Many swimmers in the 2008 Beijing Olympics wore (Speedo) body suits; it might have made a difference in breaking many world records (**Figure 6.31**). Most elite swimmers (and cyclists) shave their body hair. Such innovations can have the effect of slicing away milliseconds in a race, sometimes making the difference between a gold and a silver medal. One consequence is that careful and precise guidelines must be continuously developed to maintain the integrity of the sport.

Figure 6.31 Body suits, such as this LZR Racer Suit, have been credited with aiding in many world records after their release in 2008. Smoother "skin" and more compression forces on a swimmer's body provide at least 10% less drag. (credit: NASA/Kathy Barnstorff)

Terminal Velocity

Some interesting situations connected to Newton's second law occur when considering the effects of drag forces upon a moving object. For instance, consider a skydiver falling through air under the influence of gravity. The two forces acting on him are the force of gravity and the drag force (ignoring the small buoyant force). The downward force of gravity remains constant regardless of the velocity at which the person is moving. However, as the person's velocity increases, the magnitude of the drag force increases until the magnitude of the drag force is equal to the gravitational force, thus producing a net force of zero. A zero net force means that there is no acceleration, as shown by Newton's second law. At this point, the person's velocity remains constant and we say that the person has reached his **terminal velocity** (v_{T}). Since F_{D} is proportional to the speed squared, a heavier skydiver must go faster for F_{D} to equal his weight. Let's see how this works out more quantitatively.

At the terminal velocity,

$$F_{\text{net}} = mg - F_{\text{D}} = ma = 0.$$

Thus,

$$mg = F_{\text{D}}.$$

Using the equation for drag force, we have

$$mg = \frac{1}{2}C\rho A v_{\text{T}}^2.$$

Solving for the velocity, we obtain

$$v_{\text{T}} = \sqrt{\frac{2mg}{\rho CA}}.$$

Assume the density of air is $\rho = 1.21\,\text{kg/m}^3$. A 75-kg skydiver descending head first has a cross-sectional area of approximately $A = 0.18\,\text{m}^2$ and a drag coefficient of approximately $C = 0.70$. We find that

$$v_\text{T} = \sqrt{\frac{2(75\,\text{kg})(9.80\,\text{m/s}^2)}{(1.21\,\text{kg/m}^3)(0.70)(0.18\,\text{m}^2)}} = 98\,\text{m/s} = 350\,\text{km/h}.$$

This means a skydiver with a mass of 75 kg achieves a terminal velocity of about 350 km/h while traveling in a pike (head first) position, minimizing the area and his drag. In a spread-eagle position, that terminal velocity may decrease to about 200 km/h as the area increases. This terminal velocity becomes much smaller after the parachute opens.

Example 6.17

Terminal Velocity of a Skydiver

Find the terminal velocity of an 85-kg skydiver falling in a spread-eagle position.

Strategy

At terminal velocity, $F_\text{net} = 0.$ Thus, the drag force on the skydiver must equal the force of gravity (the person's weight). Using the equation of drag force, we find $mg = \frac{1}{2}\rho C A v^2.$

Solution

The terminal velocity v_T can be written as

$$v_\text{T} = \sqrt{\frac{2mg}{\rho CA}} = \sqrt{\frac{2(85\,\text{kg})(9.80\,\text{m/s}^2)}{(1.21\,\text{kg/m}^3)(1.0)(0.70\,\text{m}^2)}} = 44\,\text{m/s}.$$

Significance

This result is consistent with the value for v_T mentioned earlier. The 75-kg skydiver going feet first had a terminal velocity of $v_\text{T} = 98\,\text{m/s}.$ He weighed less but had a smaller frontal area and so a smaller drag due to the air.

6.10 Check Your Understanding Find the terminal velocity of a 50-kg skydiver falling in spread-eagle fashion.

The size of the object that is falling through air presents another interesting application of air drag. If you fall from a 5-m-high branch of a tree, you will likely get hurt—possibly fracturing a bone. However, a small squirrel does this all the time, without getting hurt. You do not reach a terminal velocity in such a short distance, but the squirrel does.

The following interesting quote on animal size and terminal velocity is from a 1928 essay by a British biologist, J. B. S. Haldane, titled "On Being the Right Size."

"To the mouse and any smaller animal, [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, and a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force."

The above quadratic dependence of air drag upon velocity does not hold if the object is very small, is going very slow, or is in a denser medium than air. Then we find that the drag force is proportional just to the velocity. This relationship is given by Stokes' law.

Stokes' Law

For a spherical object falling in a medium, the drag force is

$$F_s = 6\pi r\eta v, \tag{6.6}$$

where r is the radius of the object, η is the viscosity of the fluid, and v is the object's velocity.

Good examples of Stokes' law are provided by microorganisms, pollen, and dust particles. Because each of these objects is so small, we find that many of these objects travel unaided only at a constant (terminal) velocity. Terminal velocities for bacteria (size about $1\ \mu m$) can be about $2\ \mu m/s$. To move at a greater speed, many bacteria swim using flagella (organelles shaped like little tails) that are powered by little motors embedded in the cell.

Sediment in a lake can move at a greater terminal velocity (about $5\ \mu m/s$), so it can take days for it to reach the bottom of the lake after being deposited on the surface.

If we compare animals living on land with those in water, you can see how drag has influenced evolution. Fish, dolphins, and even massive whales are streamlined in shape to reduce drag forces. Birds are streamlined and migratory species that fly large distances often have particular features such as long necks. Flocks of birds fly in the shape of a spearhead as the flock forms a streamlined pattern (**Figure 6.32**). In humans, one important example of streamlining is the shape of sperm, which need to be efficient in their use of energy.

Figure 6.32 Geese fly in a V formation during their long migratory travels. This shape reduces drag and energy consumption for individual birds, and also allows them a better way to communicate. (credit: "Julo"/Wikimedia Commons)

In lecture demonstrations, we do **measurements of the drag force (https://openstax.org/l/21dragforce)** on different objects. The objects are placed in a uniform airstream created by a fan. Calculate the Reynolds number and the drag coefficient.

The Calculus of Velocity-Dependent Frictional Forces

When a body slides across a surface, the frictional force on it is approximately constant and given by $\mu_k N$. Unfortunately, the frictional force on a body moving through a liquid or a gas does not behave so simply. This drag force is generally a complicated function of the body's velocity. However, for a body moving in a straight line at moderate speeds through a liquid such as water, the frictional force can often be approximated by

$$f_R = -bv,$$

where b is a constant whose value depends on the dimensions and shape of the body and the properties of the liquid, and v is the velocity of the body. Two situations for which the frictional force can be represented this equation are a motorboat moving through water and a small object falling slowly through a liquid.

Let's consider the object falling through a liquid. The free-body diagram of this object with the positive direction downward is shown in **Figure 6.33**. Newton's second law in the vertical direction gives the differential equation

$$mg - bv = m\frac{dv}{dt},$$

where we have written the acceleration as dv/dt. As v increases, the frictional force $-bv$ increases until it matches mg. At this point, there is no acceleration and the velocity remains constant at the terminal velocity v_{T}. From the previous equation,

$$mg - bv_{\text{T}} = 0,$$

so

$$v_{\text{T}} = \frac{mg}{b}.$$

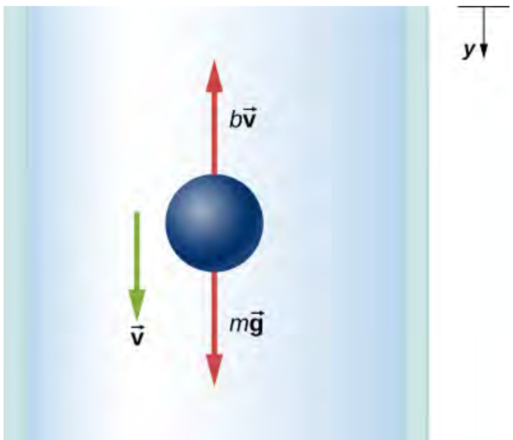


Figure 6.33 Free-body diagram of an object falling through a resistive medium.

We can find the object's velocity by integrating the differential equation for v. First, we rearrange terms in this equation to obtain

$$\frac{dv}{g - (b/m)v} = dt.$$

Assuming that $v = 0$ at $t = 0$, integration of this equation yields

$$\int_0^v \frac{dv'}{g - (b/m)v'} = \int_0^t dt',$$

or

$$-\frac{m}{b}\ln\left(g - \frac{b}{m}v'\right)\Big|_0^v = t'|_0^t,$$

where v' and t' are dummy variables of integration. With the limits given, we find

$$-\frac{m}{b}[\ln\left(g - \frac{b}{m}v\right) - \ln g] = t.$$

Since $\ln A - \ln B = \ln(A/B)$, and $\ln(A/B) = x$ implies $e^x = A/B$, we obtain

$$\frac{g - (bv/m)}{g} = e^{-bt/m},$$

and

$$v = \frac{mg}{b}(1 - e^{-bt/m}).$$

Notice that as $t \to \infty$, $v \to mg/b = v_{\text{T}}$, which is the terminal velocity.

The position at any time may be found by integrating the equation for v. With $v = dy/dt$,

$$dy = \frac{mg}{b}(1 - e^{-bt/m})dt.$$

Assuming $y = 0$ when $t = 0$,

$$\int_0^y dy' = \frac{mg}{b}\int_0^t (1 - e^{-bt'/m})dt',$$

which integrates to

$$y = \frac{mg}{b}t + \frac{m^2 g}{b^2}(e^{-bt/m} - 1).$$

Example 6.18

Effect of the Resistive Force on a Motorboat

A motorboat is moving across a lake at a speed v_0 when its motor suddenly freezes up and stops. The boat then slows down under the frictional force $f_R = -bv.$ (a) What are the velocity and position of the boat as functions of time? (b) If the boat slows down from 4.0 to 1.0 m/s in 10 s, how far does it travel before stopping?

Solution

a. With the motor stopped, the only horizontal force on the boat is $f_R = -bv,$ so from Newton's second law,

$$m\frac{dv}{dt} = -bv,$$

which we can write as

$$\frac{dv}{v} = -\frac{b}{m}dt.$$

Integrating this equation between the time zero when the velocity is v_0 and the time t when the velocity is v, we have

$$\int_0^v \frac{dv'}{v'} = -\frac{b}{m}\int_0^t dt'.$$

Thus,

$$\ln\frac{v}{v_0} = -\frac{b}{m}t,$$

which, since $\ln A = x$ implies $e^x = A,$ we can write this as

$$v = v_0 e^{-bt/m}.$$

Now from the definition of velocity,

$$\frac{dx}{dt} = v_0 e^{-bt/m},$$

so we have

$$dx = v_0 e^{-bt/m} dt.$$

With the initial position zero, we have

$$\int_0^x dx' = v_0 \int_0^t e^{-bt'/m} dt',$$

and

$$x = -\frac{mv_0}{b} e^{-bt'/m} \Big|_0^t = \frac{mv_0}{b}(1 - e^{-bt/m}).$$

As time increases, $e^{-bt/m} \to 0,$ and the position of the boat approaches a limiting value

$$x_{\text{max}} = \frac{mv_0}{b}.$$

Although this tells us that the boat takes an infinite amount of time to reach $x_{\text{max}},$ the boat effectively stops after a reasonable time. For example, at $t = 10m/b,$ we have

$$v = v_0 e^{-10} \simeq 4.5 \times 10^{-5} v_0,$$

whereas we also have

$$x = x_{\text{max}}(1 - e^{-10}) \simeq 0.99995 x_{\text{max}}.$$

Therefore, the boat's velocity and position have essentially reached their final values.

b. With $v_0 = 4.0\text{ m/s}$ and $v = 1.0\text{ m/s},$ we have $1.0\text{ m/s} = (4.0\text{ m/s})e^{-(b/m)(10\text{ s})},$ so

$$\ln 0.25 = -\ln 4.0 = -\frac{b}{m}(10\text{ s}),$$

and

$$\frac{b}{m} = \frac{1}{10}\ln 4.0\text{ s}^{-1} = 0.14\text{ s}^{-1}.$$

Now the boat's limiting position is

$$x_{\text{max}} = \frac{mv_0}{b} = \frac{4.0\text{ m/s}}{0.14\text{ s}^{-1}} = 29\text{ m}.$$

Significance

In the both of the previous examples, we found "limiting" values. The terminal velocity is the same as the limiting velocity, which is the velocity of the falling object after a (relatively) long time has passed. Similarly, the limiting distance of the boat is the distance the boat will travel after a long amount of time has passed. Due to the properties of exponential decay, the time involved to reach either of these values is actually not too long (certainly not an infinite amount of time!) but they are quickly found by taking the limit to infinity.

6.11 Check Your Understanding Suppose the resistive force of the air on a skydiver can be approximated by $f = -bv^2$. If the terminal velocity of a 100-kg skydiver is 60 m/s, what is the value of b?

CHAPTER 6 REVIEW

KEY TERMS

banked curve curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

centripetal force any net force causing uniform circular motion

Coriolis force inertial force causing the apparent deflection of moving objects when viewed in a rotating frame of reference

drag force force that always opposes the motion of an object in a fluid; unlike simple friction, the drag force is proportional to some function of the velocity of the object in that fluid

friction force that opposes relative motion or attempts at motion between systems in contact

ideal banking sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

inertial force force that has no physical origin

kinetic friction force that opposes the motion of two systems that are in contact and moving relative to each other

noninertial frame of reference accelerated frame of reference

static friction force that opposes the motion of two systems that are in contact and are not moving relative to each other

terminal velocity constant velocity achieved by a falling object, which occurs when the weight of the object is balanced by the upward drag force

KEY EQUATIONS

Magnitude of static friction	$f_s \leq \mu_s N$
Magnitude of kinetic friction	$f_k = \mu_k N$
Centripetal force	$F_c = m\frac{v^2}{r}$ or $F_c = mr\omega^2$
Ideal angle of a banked curve	$\tan\theta = \frac{v^2}{rg}$
Drag force	$F_D = \frac{1}{2}C\rho A v^2$
Stokes' law	$F_s = 6\pi r\eta v$

SUMMARY

6.1 Solving Problems with Newton's Laws

- Newton's laws of motion can be applied in numerous situations to solve motion problems.
- Some problems contain multiple force vectors acting in different directions on an object. Be sure to draw diagrams, resolve all force vectors into horizontal and vertical components, and draw a free-body diagram. Always analyze the direction in which an object accelerates so that you can determine whether $F_{\text{net}} = ma$ or $F_{\text{net}} = 0$.
- The normal force on an object is not always equal in magnitude to the weight of the object. If an object is accelerating vertically, the normal force is less than or greater than the weight of the object. Also, if the object is on an inclined plane, the normal force is always less than the full weight of the object.
- Some problems contain several physical quantities, such as forces, acceleration, velocity, or position. You can apply concepts from kinematics and dynamics to solve these problems.

6.2 Friction

- Friction is a contact force that opposes the motion or attempted motion between two systems. Simple friction is proportional to the normal force N supporting the two systems.
- The magnitude of static friction force between two materials stationary relative to each other is determined using the coefficient of static friction, which depends on both materials.
- The kinetic friction force between two materials moving relative to each other is determined using the coefficient of kinetic friction, which also depends on both materials and is always less than the coefficient of static friction.

6.3 Centripetal Force

- Centripetal force $\vec{\mathbf{F}}_{\text{c}}$ is a "center-seeking" force that always points toward the center of rotation. It is perpendicular to linear velocity and has the magnitude
$$F_{\text{c}} = ma_{\text{c}}.$$
- Rotating and accelerated frames of reference are noninertial. Inertial forces, such as the Coriolis force, are needed to explain motion in such frames.

6.4 Drag Force and Terminal Speed

- Drag forces acting on an object moving in a fluid oppose the motion. For larger objects (such as a baseball) moving at a velocity in air, the drag force is determined using the drag coefficient (typical values are given in **Table 6.2**), the area of the object facing the fluid, and the fluid density.
- For small objects (such as a bacterium) moving in a denser medium (such as water), the drag force is given by Stokes' law.

CONCEPTUAL QUESTIONS

6.1 Solving Problems with Newton's Laws

1. To simulate the apparent weightlessness of space orbit, astronauts are trained in the hold of a cargo aircraft that is accelerating downward at g. Why do they appear to be weightless, as measured by standing on a bathroom scale, in this accelerated frame of reference? Is there any difference between their apparent weightlessness in orbit and in the aircraft?

6.2 Friction

2. The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

3. When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

4. When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular, explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

5. A physics major is cooking breakfast when she notices that the frictional force between her steel spatula and Teflon frying pan is only 0.200 N. Knowing the coefficient of kinetic friction between the two materials, she quickly calculates the normal force. What is it?

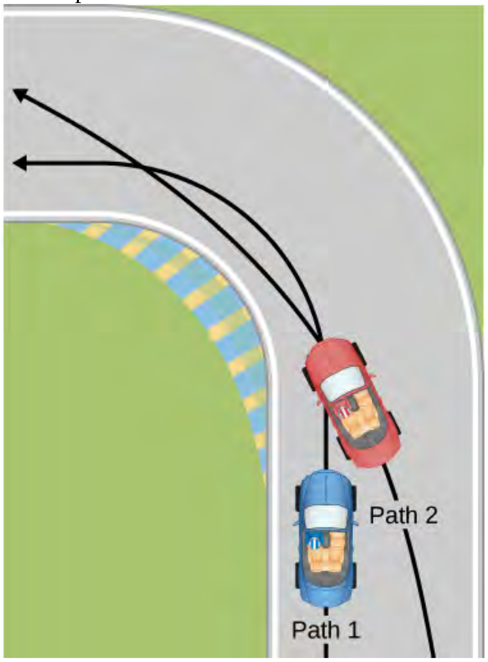
6.3 Centripetal Force

6. If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

7. Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

8. If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.

9. Race car drivers routinely cut corners, as shown below (Path 2). Explain how this allows the curve to be taken at the greatest speed.



10. Many amusement parks have rides that make vertical loops like the one shown below. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

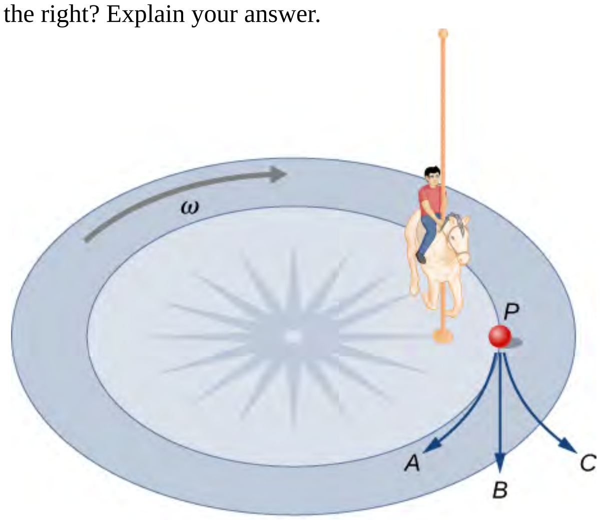
(a) The car goes over the top at faster than this speed?

(b) The car goes over the top at slower than this speed?

11. What causes water to be removed from clothes in a spin-dryer?

12. As a skater forms a circle, what force is responsible for making his turn? Use a free-body diagram in your answer.

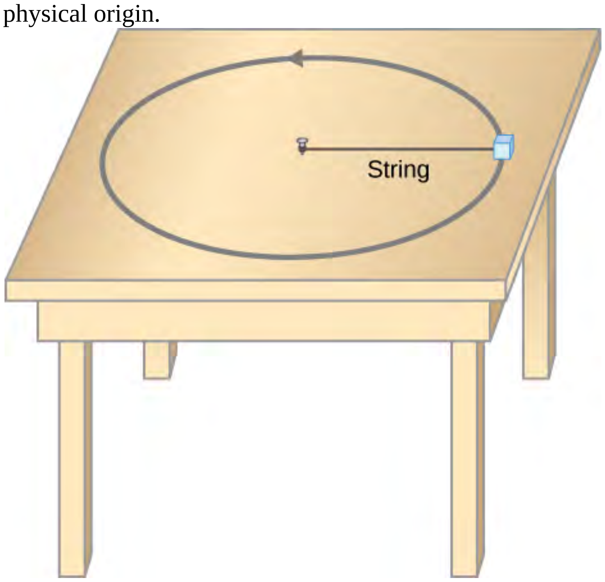
13. Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown below will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.



Merry-go-round's rotating frame of reference

14. Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

15. Suppose a mass is moving in a circular path on a frictionless table as shown below. In Earth's frame of reference, there is no centrifugal force pulling the mass away from the center of rotation, yet there is a force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.



16. When a toilet is flushed or a sink is drained, the water (and other material) begins to rotate about the drain on the way down. Assuming no initial rotation and a flow initially directly straight toward the drain, explain what causes the rotation and which direction it has in the Northern Hemisphere. (Note that this is a small effect and in most toilets the rotation is caused by directional water jets.) Would the direction of rotation reverse if water were forced up the drain?

17. A car rounds a curve and encounters a patch of ice with a very low coefficient of kinetic fiction. The car slides off the road. Describe the path of the car as it leaves the road.

18. In one amusement park ride, riders enter a large vertical barrel and stand against the wall on its horizontal floor. The barrel is spun up and the floor drops away. Riders feel as if they are pinned to the wall by a force something like the gravitational force. This is an inertial force sensed and used by the riders to explain events in the rotating frame of reference of the barrel. Explain in an inertial frame of reference (Earth is nearly one) what pins the riders to the wall, and identify all forces acting on them.

19. Two friends are having a conversation. Anna says a satellite in orbit is in free fall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in free fall because the acceleration due to gravity is not $9.80\ \text{m/s}^2$. Who do you agree with and why?

20. A nonrotating frame of reference placed at the center of the Sun is very nearly an inertial one. Why is it not exactly an inertial frame?

6.4 Drag Force and Terminal Speed

21. Athletes such as swimmers and bicyclists wear body suits in competition. Formulate a list of pros and cons of such suits.

22. Two expressions were used for the drag force experienced by a moving object in a liquid. One depended upon the speed, while the other was proportional to the square of the speed. In which types of motion would each of these expressions be more applicable than the other one?

23. As cars travel, oil and gasoline leaks onto the road surface. If a light rain falls, what does this do to the control of the car? Does a heavy rain make any difference?

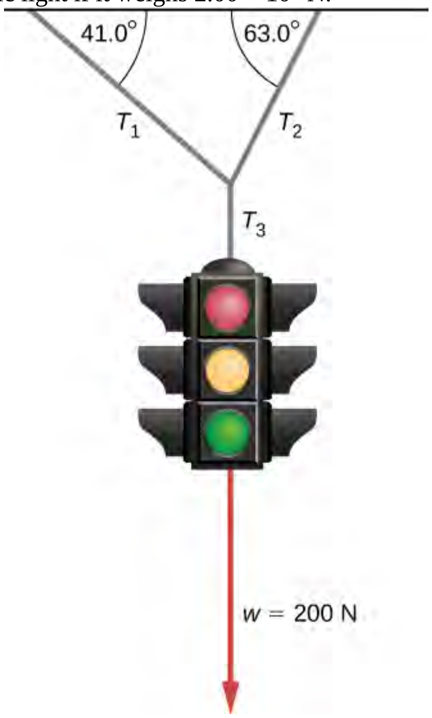
24. Why can a squirrel jump from a tree branch to the ground and run away undamaged, while a human could break a bone in such a fall?

PROBLEMS

6.1 Solving Problems with Newton's Laws

25. A 30.0-kg girl in a swing is pushed to one side and held at rest by a horizontal force $\vec{\mathbf{F}}$ so that the swing ropes are $30.0°$ with respect to the vertical. (a) Calculate the tension in each of the two ropes supporting the swing under these conditions. (b) Calculate the magnitude of $\vec{\mathbf{F}}$.

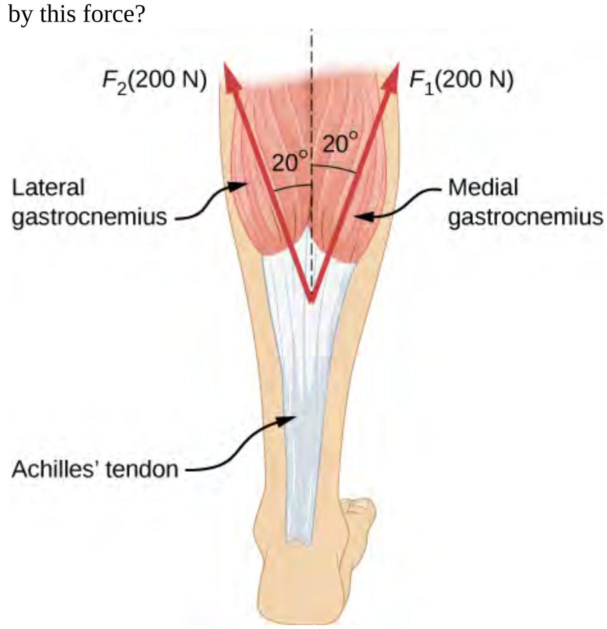
26. Find the tension in each of the three cables supporting the traffic light if it weighs 2.00×10^2 N.



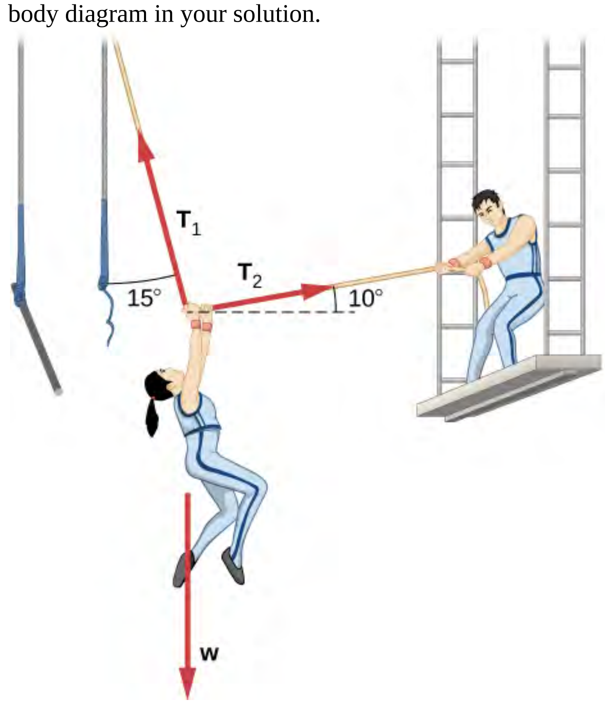
27. Three forces act on an object, considered to be a particle, which moves with constant velocity $v = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$ m/s. Two of the forces are $\vec{\mathbf{F}}_1 = (3\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}})$ N and $\vec{\mathbf{F}}_2 = (4\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ N. Find the third force.

28. A flea jumps by exerting a force of 1.20×10^{-5} N straight down on the ground. A breeze blowing on the flea parallel to the ground exerts a force of 0.500×10^{-6} N on the flea while the flea is still in contact with the ground. Find the direction and magnitude of the acceleration of the flea if its mass is 6.00×10^{-7} kg. Do not neglect the gravitational force.

29. Two muscles in the back of the leg pull upward on the Achilles tendon, as shown below. (These muscles are called the medial and lateral heads of the gastrocnemius muscle.) Find the magnitude and direction of the total force on the Achilles tendon. What type of movement could be caused by this force?



30. After a mishap, a 76.0-kg circus performer clings to a trapeze, which is being pulled to the side by another circus artist, as shown here. Calculate the tension in the two ropes if the person is momentarily motionless. Include a free-body diagram in your solution.



31. A 35.0-kg dolphin decelerates from 12.0 to 7.50 m/s in 2.30 s to join another dolphin in play. What average force was exerted to slow the first dolphin if it was moving horizontally? (The gravitational force is balanced by the buoyant force of the water.)

32. When starting a foot race, a 70.0-kg sprinter exerts an average force of 650 N backward on the ground for 0.800 s. (a) What is his final speed? (b) How far does he travel?

33. A large rocket has a mass of 2.00×10^6 kg at takeoff, and its engines produce a thrust of 3.50×10^7 N. (a) Find its initial acceleration if it takes off vertically. (b) How long does it take to reach a velocity of 120 km/h straight up, assuming constant mass and thrust?

34. A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110.0 kg.

35. A 2.50-kg fireworks shell is fired straight up from a mortar and reaches a height of 110.0 m. (a) Neglecting air resistance (a poor assumption, but we will make it for this example), calculate the shell’s velocity when it leaves the mortar. (b) The mortar itself is a tube 0.450 m long. Calculate the average acceleration of the shell in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the shell in the mortar? Express your answer in newtons and as a ratio to the weight of the shell.

36. A 0.500-kg potato is fired at an angle of $80.0°$ above the horizontal from a PVC pipe used as a “potato gun” and reaches a height of 110.0 m. (a) Neglecting air resistance, calculate the potato’s velocity when it leaves the gun. (b) The gun itself is a tube 0.450 m long. Calculate the average acceleration of the potato in the tube as it goes from zero to the velocity found in (a). (c) What is the average force on the potato in the gun? Express your answer in newtons and as a ratio to the weight of the potato.

37. An elevator filled with passengers has a mass of 1.70×10^3 kg. (a) The elevator accelerates upward from rest at a rate of $1.20\ \text{m/s}^2$ for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of $0.600\ \text{m/s}^2$ for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?

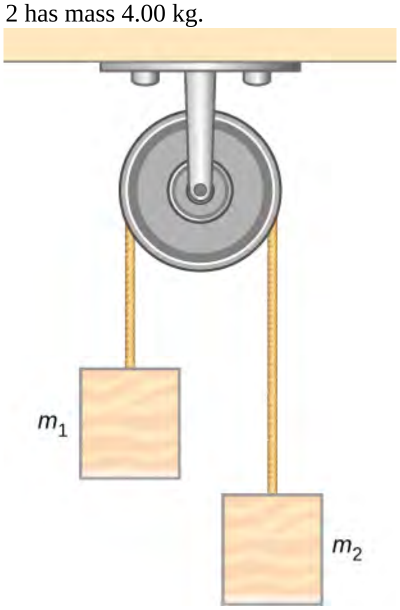
38. A 20.0-g ball hangs from the roof of a freight car by a string. When the freight car begins to move, the string makes an angle of $35.0°$ with the vertical. (a) What is the acceleration of the freight car? (b) What is the tension in the string?

39. A student’s backpack, full of textbooks, is hung from a spring scale attached to the ceiling of an elevator. When the elevator is accelerating downward at $3.8\ \text{m/s}^2$, the scale reads 60 N. (a) What is the mass of the backpack? (b) What does the scale read if the elevator moves upward while slowing down at a rate $3.8\ \text{m/s}^2$? (c) What does the scale read if the elevator moves upward at constant velocity? (d) If the elevator had no brakes and the cable supporting it were to break loose so that the elevator could fall freely, what would the spring scale read?

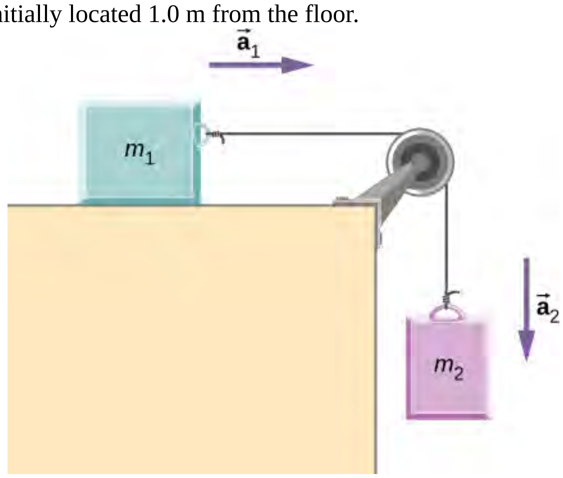
40. A service elevator takes a load of garbage, mass 10.0 kg, from a floor of a skyscraper under construction, down to ground level, accelerating downward at a rate of $1.2\ \text{m/s}^2$. Find the magnitude of the force the garbage exerts on the floor of the service elevator?

41. A roller coaster car starts from rest at the top of a track 30.0 m long and inclined at $20.0°$ to the horizontal. Assume that friction can be ignored. (a) What is the acceleration of the car? (b) How much time elapses before it reaches the bottom of the track?

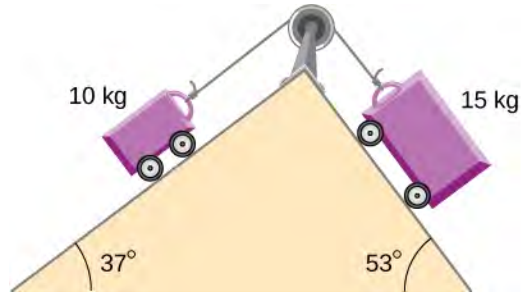
42. The device shown below is the Atwood's machine considered in Example 6.5. Assuming that the masses of the string and the frictionless pulley are negligible, (a) find an equation for the acceleration of the two blocks; (b) find an equation for the tension in the string; and (c) find both the acceleration and tension when block 1 has mass 2.00 kg and block 2 has mass 4.00 kg.



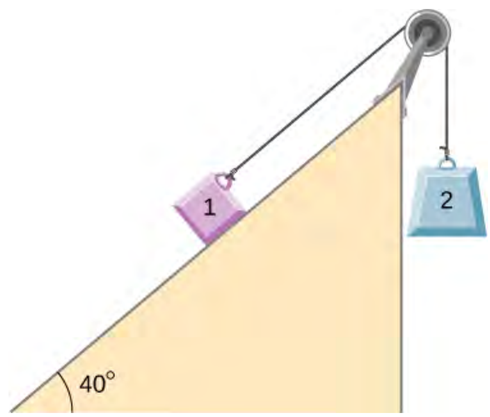
43. Two blocks are connected by a massless rope as shown below. The mass of the block on the table is 4.0 kg and the hanging mass is 1.0 kg. The table and the pulley are frictionless. (a) Find the acceleration of the system. (b) Find the tension in the rope. (c) Find the speed with which the hanging mass hits the floor if it starts from rest and is initially located 1.0 m from the floor.



44. Shown below are two carts connected by a cord that passes over a small frictionless pulley. Each cart rolls freely with negligible friction. Calculate the acceleration of the carts and the tension in the cord.



45. A 2.00 kg block (mass 1) and a 4.00 kg block (mass 2) are connected by a light string as shown; the inclination of the ramp is $40.0°$. Friction is negligible. What is (a) the acceleration of each block and (b) the tension in the string?



6.2 Friction

46. (a) When rebuilding his car's engine, a physics major must exert 3.00×10^2 N of force to insert a dry steel piston into a steel cylinder. What is the normal force between the piston and cylinder? (b) What force would he have to exert if the steel parts were oiled?

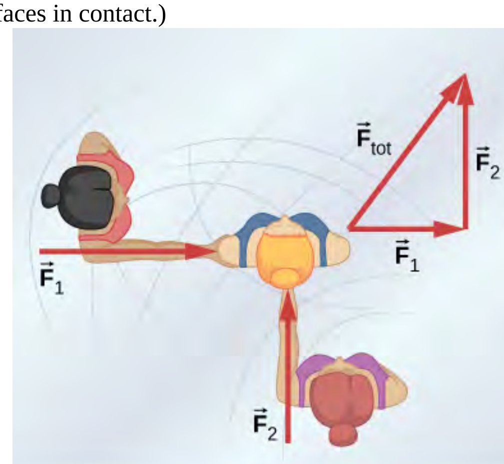
47. (a) What is the maximum frictional force in the knee joint of a person who supports 66.0 kg of her mass on that knee? (b) During strenuous exercise, it is possible to exert forces to the joints that are easily 10 times greater than the weight being supported. What is the maximum force of friction under such conditions? The frictional forces in joints are relatively small in all circumstances except when the joints deteriorate, such as from injury or arthritis. Increased frictional forces can cause further damage and pain.

48. Suppose you have a 120-kg wooden crate resting on a wood floor, with coefficient of static friction 0.500 between these wood surfaces. (a) What maximum force can you exert horizontally on the crate without moving it? (b) If you continue to exert this force once the crate starts to slip, what will its acceleration then be? The coefficient of sliding friction is known to be 0.300 for this situation.

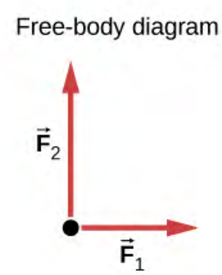
49. (a) If half of the weight of a small 1.00×10^3-kg utility truck is supported by its two drive wheels, what is the maximum acceleration it can achieve on dry concrete? (b) Will a metal cabinet lying on the wooden bed of the truck slip if it accelerates at this rate? (c) Solve both problems assuming the truck has four-wheel drive.

50. A team of eight dogs pulls a sled with waxed wood runners on wet snow (mush!). The dogs have average masses of 19.0 kg, and the loaded sled with its rider has a mass of 210 kg. (a) Calculate the acceleration of the dogs starting from rest if each dog exerts an average force of 185 N backward on the snow. (b) Calculate the force in the coupling between the dogs and the sled.

51. Consider the 65.0-kg ice skater being pushed by two others shown below. (a) Find the direction and magnitude of $\mathbf{F}_{\text{tot}}$, the total force exerted on her by the others, given that the magnitudes F_1 and F_2 are 26.4 N and 18.6 N, respectively. (b) What is her initial acceleration if she is initially stationary and wearing steel-bladed skates that point in the direction of $\mathbf{F}_{\text{tot}}$? (c) What is her acceleration assuming she is already moving in the direction of $\mathbf{F}_{\text{tot}}$? (Remember that friction always acts in the direction opposite that of motion or attempted motion between surfaces in contact.)

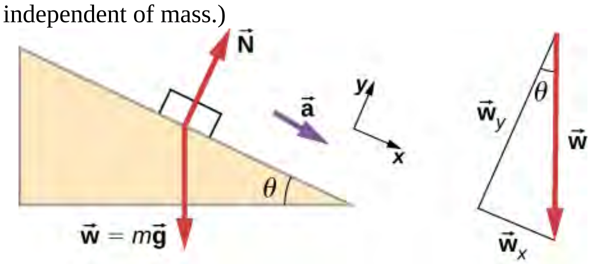


(a)

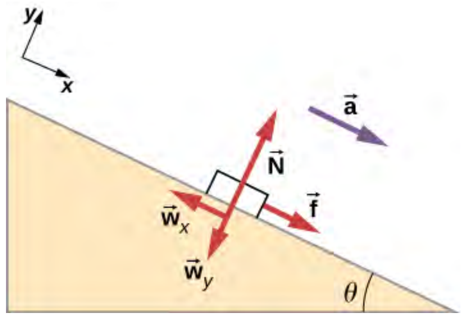


(b)

52. Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g \sin\theta$. (Note that this acceleration is independent of mass.)



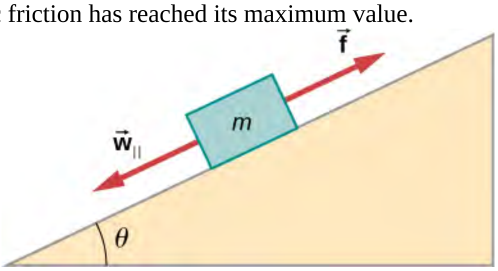
53. Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_k = \mu_k N$) is $a = g(\sin\theta - \mu_k \cos\theta)$. Note that the acceleration is independent of mass and reduces to the expression found in the previous problem when friction becomes negligibly small $(\mu_k = 0)$.



54. Calculate the deceleration of a snow boarder going up a $5.00°$ slope, assuming the coefficient of friction for waxed wood on wet snow. The result of the preceding problem may be useful, but be careful to consider the fact that the snow boarder is going uphill.

55. A machine at a post office sends packages out a chute and down a ramp to be loaded into delivery vehicles. (a) Calculate the acceleration of a box heading down a $10.0°$ slope, assuming the coefficient of friction for a parcel on waxed wood is 0.100. (b) Find the angle of the slope down which this box could move at a constant velocity. You can neglect air resistance in both parts.

56. If an object is to rest on an incline without slipping, then friction must equal the component of the weight of the object parallel to the incline. This requires greater and greater friction for steeper slopes. Show that the maximum angle of an incline above the horizontal for which an object will not slide down is $\theta = \tan^{-1} \mu_s$. You may use the result of the previous problem. Assume that $a = 0$ and that static friction has reached its maximum value.



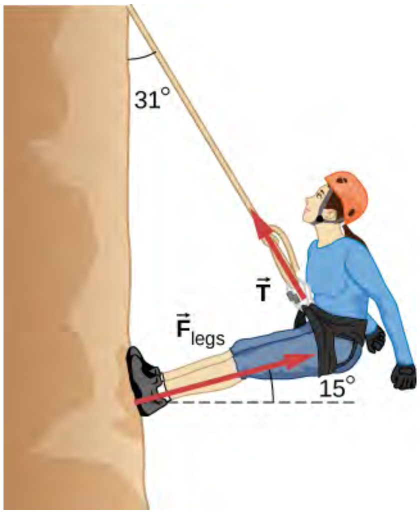
57. Calculate the maximum acceleration of a car that is heading down a $6.00°$ slope (one that makes an angle of $6.00°$ with the horizontal) under the following road conditions. You may assume that the weight of the car is evenly distributed on all four tires and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the deceleration. (Ignore rolling.) Calculate for a car: (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

58. Calculate the maximum acceleration of a car that is heading up a $4.00°$ slope (one that makes an angle of $4.00°$ with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that $\mu_s = 0.100$, the same as for shoes on ice.

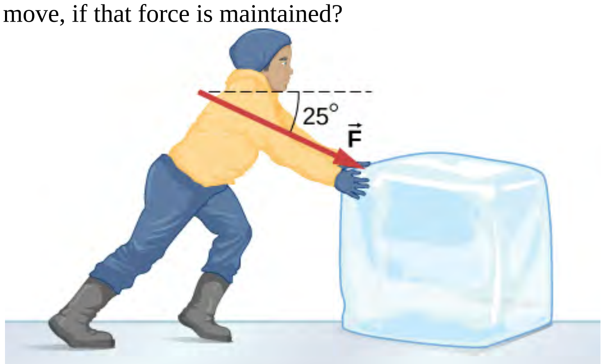
59. Repeat the preceding problem for a car with four-wheel drive.

60. A freight train consists of two 8.00×10^5-kg engines and 45 cars with average masses of 5.50×10^5 kg. (a) What force must each engine exert backward on the track to accelerate the train at a rate of $5.00 \times 10^{-2}\,\text{m/s}^2$ if the force of friction is $7.50 \times 10^5\,\text{N}$, assuming the engines exert identical forces? This is not a large frictional force for such a massive system. Rolling friction for trains is small, and consequently, trains are very energy-efficient transportation systems. (b) What is the force in the coupling between the 37th and 38th cars (this is the force each exerts on the other), assuming all cars have the same mass and that friction is evenly distributed among all of the cars and engines?

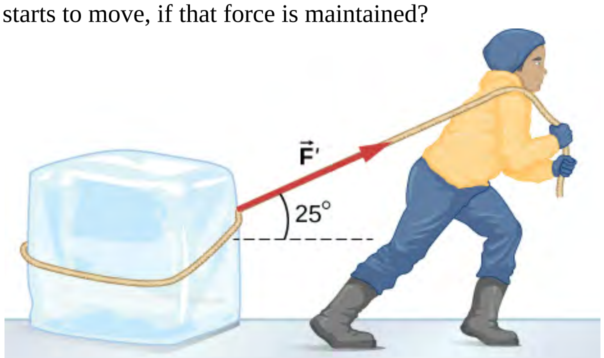
61. Consider the 52.0-kg mountain climber shown below. (a) Find the tension in the rope and the force that the mountain climber must exert with her feet on the vertical rock face to remain stationary. Assume that the force is exerted parallel to her legs. Also, assume negligible force exerted by her arms. (b) What is the minimum coefficient of friction between her shoes and the cliff?



62. A contestant in a winter sporting event pushes a 45.0-kg block of ice across a frozen lake as shown below. (a) Calculate the minimum force *F* he must exert to get the block moving. (b) What is its acceleration once it starts to move, if that force is maintained?



63. The contestant now pulls the block of ice with a rope over his shoulder at the same angle above the horizontal as shown below. Calculate the minimum force *F* he must exert to get the block moving. (b) What is its acceleration once it starts to move, if that force is maintained?



64. At a post office, a parcel that is a 20.0-kg box slides down a ramp inclined at $30.0°$ with the horizontal. The coefficient of kinetic friction between the box and plane is 0.0300. (a) Find the acceleration of the box. (b) Find the velocity of the box as it reaches the end of the plane, if the length of the plane is 2 m and the box starts at rest.

6.3 Centripetal Force

65. (a) A 22.0-kg child is riding a playground merry-go-round that is rotating at 40.0 rev/min. What centripetal force is exerted if he is 1.25 m from its center? (b) What centripetal force is exerted if the merry-go-round rotates at 3.00 rev/min and he is 8.00 m from its center? (c) Compare each force with his weight.

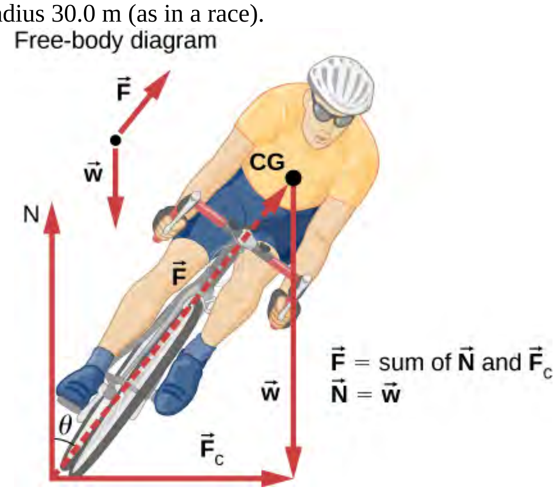
66. Calculate the centripetal force on the end of a 100-m (radius) wind turbine blade that is rotating at 0.5 rev/s. Assume the mass is 4 kg.

67. What is the ideal banking angle for a gentle turn of 1.20-km radius on a highway with a 105 km/h speed limit (about 65 mi/h), assuming everyone travels at the limit?

68. What is the ideal speed to take a 100.0-m-radius curve banked at a $20.0°$ angle?

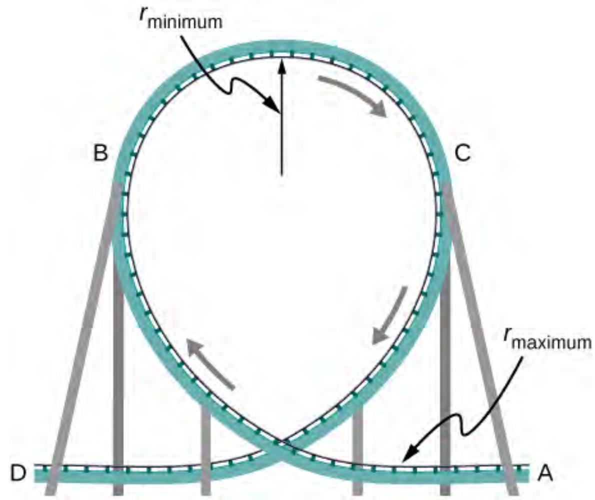
69. (a) What is the radius of a bobsled turn banked at $75.0°$ and taken at 30.0 m/s, assuming it is ideally banked? (b) Calculate the centripetal acceleration. (c) Does this acceleration seem large to you?

70. Part of riding a bicycle involves leaning at the correct angle when making a turn, as seen below. To be stable, the force exerted by the ground must be on a line going through the center of gravity. The force on the bicycle wheel can be resolved into two perpendicular components—friction parallel to the road (this must supply the centripetal force) and the vertical normal force (which must equal the system's weight). (a) Show that θ (as defined as shown) is related to the speed v and radius of curvature r of the turn in the same way as for an ideally banked roadway—that is, $\theta = \tan^{-1}(v^2/rg)$. (b) Calculate θ for a 12.0-m/s turn of radius 30.0 m (as in a race).

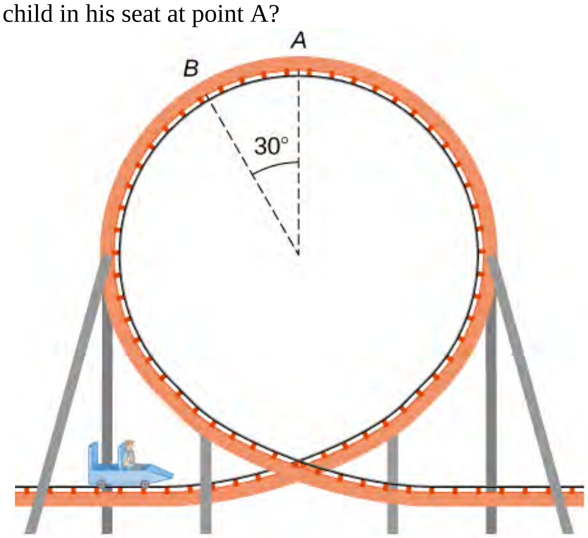


71. If a car takes a banked curve at less than the ideal speed, friction is needed to keep it from sliding toward the inside of the curve (a problem on icy mountain roads). (a) Calculate the ideal speed to take a 100.0 m radius curve banked at $15.0°$. (b) What is the minimum coefficient of friction needed for a frightened driver to take the same curve at 20.0 km/h?

72. Modern roller coasters have vertical loops like the one shown here. The radius of curvature is smaller at the top than on the sides so that the downward centripetal acceleration at the top will be greater than the acceleration due to gravity, keeping the passengers pressed firmly into their seats. (a) What is the speed of the roller coaster at the top of the loop if the radius of curvature there is 15.0 m and the downward acceleration of the car is 1.50 g? (b) How high above the top of the loop must the roller coaster start from rest, assuming negligible friction? (c) If it actually starts 5.00 m higher than your answer to (b), how much energy did it lose to friction? Its mass is 1.50×10^3 kg.



73. A child of mass 40.0 kg is in a roller coaster car that travels in a loop of radius 7.00 m. At point A the speed of the car is 10.0 m/s, and at point B, the speed is 10.5 m/s. Assume the child is not holding on and does not wear a seat belt. (a) What is the force of the car seat on the child at point A? (b) What is the force of the car seat on the child at point B? (c) What minimum speed is required to keep the child in his seat at point A?



74. In the simple Bohr model of the ground state of the hydrogen atom, the electron travels in a circular orbit around a fixed proton. The radius of the orbit is 5.28×10^{-11} m, and the speed of the electron is 2.18×10^{6} m/s. The mass of an electron is 9.11×10^{-31} kg . What is the force on the electron?

75. Railroad tracks follow a circular curve of radius 500.0 m and are banked at an angle of $5.0°$. For trains of what speed are these tracks designed?

76. The CERN particle accelerator is circular with a circumference of 7.0 km. (a) What is the acceleration of the protons $(m = 1.67 \times 10^{-27}\text{ kg})$ that move around the accelerator at 5% of the speed of light? (The speed of light is $v = 3.00 \times 10^{8}$ m/s.) (b) What is the force on the protons?

77. A car rounds an unbanked curve of radius 65 m. If the coefficient of static friction between the road and car is 0.70, what is the maximum speed at which the car traverse the curve without slipping?

78. A banked highway is designed for traffic moving at 90.0 km/h. The radius of the curve is 310 m. What is the angle of banking of the highway?

6.4 Drag Force and Terminal Speed

79. The terminal velocity of a person falling in air depends upon the weight and the area of the person facing the fluid. Find the terminal velocity (in meters per second and kilometers per hour) of an 80.0-kg skydiver falling in a pike (headfirst) position with a surface area of $0.140\,\text{m}^2$.

80. A 60.0-kg and a 90.0-kg skydiver jump from an airplane at an altitude of $6.00 \times 10^{3}\,\text{m}$, both falling in the pike position. Make some assumption on their frontal areas and calculate their terminal velocities. How long will it take for each skydiver to reach the ground (assuming the time to reach terminal velocity is small)? Assume all values are accurate to three significant digits.

81. A 560-g squirrel with a surface area of $930\,\text{cm}^2$ falls from a 5.0-m tree to the ground. Estimate its terminal velocity. (Use a drag coefficient for a horizontal skydiver.) What will be the velocity of a 56-kg person hitting the ground, assuming no drag contribution in such a short distance?

82. To maintain a constant speed, the force provided by a car's engine must equal the drag force plus the force of friction of the road (the rolling resistance). (a) What are the drag forces at 70 km/h and 100 km/h for a Toyota Camry? (Drag area is $0.70\,\text{m}^2$) (b) What is the drag force at 70 km/h and 100 km/h for a Hummer H2? (Drag area is 2.44 m^2) Assume all values are accurate to three significant digits.

83. By what factor does the drag force on a car increase as it goes from 65 to 110 km/h?

84. Calculate the velocity a spherical rain drop would achieve falling from 5.00 km (a) in the absence of air drag (b) with air drag. Take the size across of the drop to be 4 mm, the density to be $1.00 \times 10^{3}\text{ kg/m}^3$, and the surface area to be πr^2 .

85. Using Stokes' law, verify that the units for viscosity are kilograms per meter per second.

86. Find the terminal velocity of a spherical bacterium (diameter $2.00\,\mu\text{m}$) falling in water. You will first need to note that the drag force is equal to the weight at terminal velocity. Take the density of the bacterium to be $1.10 \times 10^{3}\text{ kg/m}^3$.

87. Stokes' law describes sedimentation of particles in liquids and can be used to measure viscosity. Particles in liquids achieve terminal velocity quickly. One can measure the time it takes for a particle to fall a certain distance and then use Stokes' law to calculate the viscosity of the liquid. Suppose a steel ball bearing (density $7.8 \times 10^{3}\text{ kg/m}^3$, diameter 3.0 mm) is dropped in a container of motor oil. It takes 12 s to fall a distance of 0.60 m. Calculate the viscosity of the oil.

88. Suppose that the resistive force of the air on a skydiver can be approximated by $f = -bv^2$. If the terminal velocity of a 50.0-kg skydiver is 60.0 m/s, what is the value of *b*?

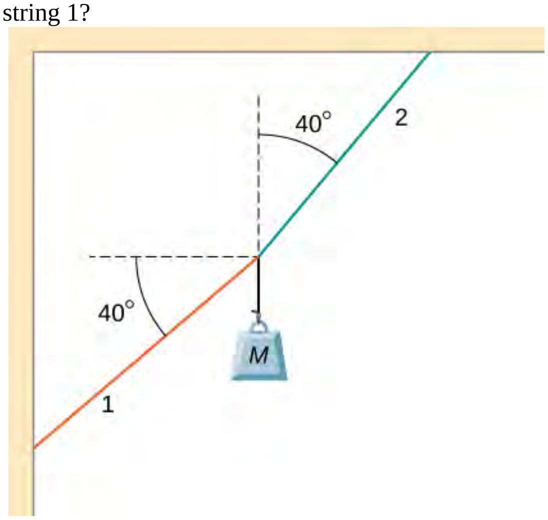
89. A small diamond of mass 10.0 g drops from a swimmer's earring and falls through the water, reaching a terminal velocity of 2.0 m/s. (a) Assuming the frictional force on the diamond obeys $f = -bv$, what is *b*? (b) How far does the diamond fall before it reaches 90 percent of its terminal speed?

90. (a) What is the final velocity of a car originally traveling at 50.0 km/h that decelerates at a rate of 0.400 m/s^2 for 50.0 s? Assume a coefficient of friction of 1.0. (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

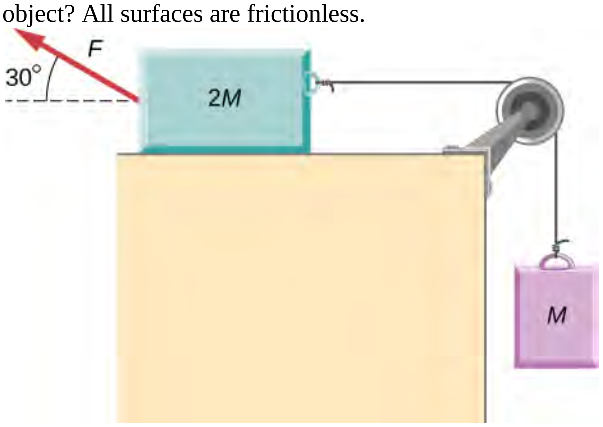
91. A 75.0-kg woman stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with her weight. (The scale exerts an upward force on her equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?

92. (a) Calculate the minimum coefficient of friction needed for a car to negotiate an unbanked 50.0 m radius curve at 30.0 m/s. (b) What is unreasonable about the result? (c) Which premises are unreasonable or inconsistent?

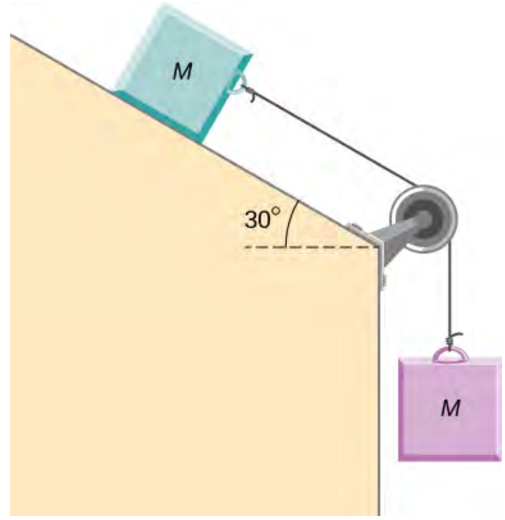
93. As shown below, if $M = 5.50 \text{ kg},$ what is the tension in string 1?



94. As shown below, if $F = 60.0 \text{ N}$ and $M = 4.00 \text{ kg},$ what is the magnitude of the acceleration of the suspended object? All surfaces are frictionless.



95. As shown below, if $M = 6.0 \text{ kg},$ what is the tension in the connecting string? The pulley and all surfaces are frictionless.



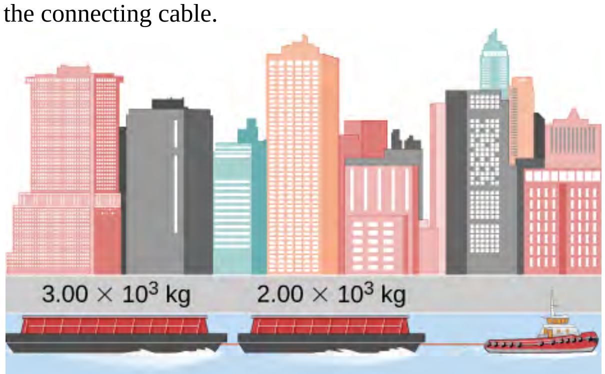
96. A small space probe is released from a spaceship. The space probe has mass 20.0 kg and contains 90.0 kg of fuel. It starts from rest in deep space, from the origin of a coordinate system based on the spaceship, and burns fuel at the rate of 3.00 kg/s. The engine provides a constant thrust of 120.0 N. (a) Write an expression for the mass of the space probe as a function of time, between 0 and 30 seconds, assuming that the engine ignites fuel beginning at $t = 0.$ (b) What is the velocity after 15.0 s? (c) What is the position of the space probe after 15.0 s, with initial position at the origin? (d) Write an expression for the position as a function of time, for $t > 30.0 \text{ s}.$

97. A half-full recycling bin has mass 3.0 kg and is pushed up a $40.0°$ incline with constant speed under the action of a 26-N force acting up and parallel to the incline. The incline has friction. What magnitude force must act up and parallel to the incline for the bin to move down the incline at constant velocity?

98. A child has mass 6.0 kg and slides down a $35°$ incline with constant speed under the action of a 34-N force acting up and parallel to the incline. What is the coefficient of kinetic friction between the child and the surface of the incline?

ADDITIONAL PROBLEMS

99. The two barges shown here are coupled by a cable of negligible mass. The mass of the front barge is 2.00×10^3 kg and the mass of the rear barge is 3.00×10^3 kg. A tugboat pulls the front barge with a horizontal force of magnitude 20.0×10^3 N, and the frictional forces of the water on the front and rear barges are 8.00×10^3 N and 10.0×10^3 N, respectively. Find the horizontal acceleration of the barges and the tension in the connecting cable.



100. If the order of the barges of the preceding exercise is reversed so that the tugboat pulls the 3.00×10^3-kg barge with a force of 20.0×10^3 N, what are the acceleration of the barges and the tension in the coupling cable?

101. An object with mass m moves along the x-axis. Its position at any time is given by $x(t) = pt^3 + qt^2$ where p and q are constants. Find the net force on this object for any time t.

102. A helicopter with mass 2.35×10^4 kg has a position given by $\vec{\mathbf{r}}(t) = (0.020\,t^3)\hat{\mathbf{i}} + (2.2t)\hat{\mathbf{j}} - (0.060\,t^2)\hat{\mathbf{k}}$. Find the net force on the helicopter at $t = 3.0$ s.

103. Located at the origin, an electric car of mass m is at rest and in equilibrium. A time dependent force of $\vec{\mathbf{F}}(t)$ is applied at time $t = 0$, and its components are $F_x(t) = p + nt$ and $F_y(t) = qt$ where p, q, and n are constants. Find the position $\vec{\mathbf{r}}(t)$ and velocity $\vec{\mathbf{v}}(t)$ as functions of time t.

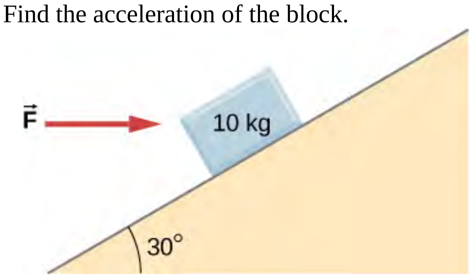
104. A particle of mass m is located at the origin. It is at rest and in equilibrium. A time-dependent force of $\vec{\mathbf{F}}(t)$ is applied at time $t = 0$, and its components are $F_x(t) = pt$ and $F_y(t) = n + qt$ where p, q, and n are constants. Find the position $\vec{\mathbf{r}}(t)$ and velocity $\vec{\mathbf{v}}(t)$ as functions of time t.

105. A 2.0-kg object has a velocity of $4.0\hat{\mathbf{i}}$ m/s at $t = 0$. A constant resultant force of $(2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})$ N then acts on the object for 3.0 s. What is the magnitude of the object's velocity at the end of the 3.0-s interval?

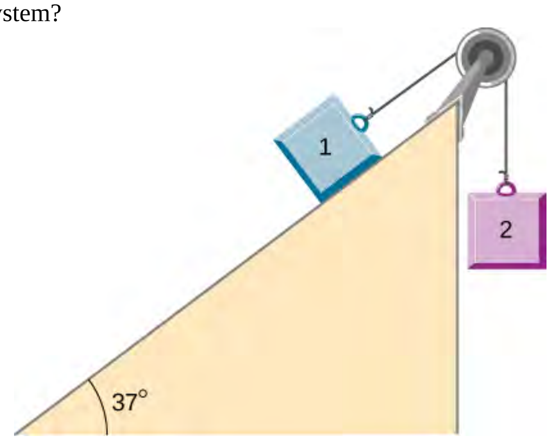
106. A 1.5-kg mass has an acceleration of $(4.0\hat{\mathbf{i}} - 3.0\hat{\mathbf{j}})$ m/s^2. Only two forces act on the mass. If one of the forces is $(2.0\hat{\mathbf{i}} - 1.4\hat{\mathbf{j}})$ N, what is the magnitude of the other force?

107. A box is dropped onto a conveyor belt moving at 3.4 m/s. If the coefficient of friction between the box and the belt is 0.27, how long will it take before the box moves without slipping?

108. Shown below is a 10.0-kg block being pushed by a horizontal force $\vec{\mathbf{F}}$ of magnitude 200.0 N. The coefficient of kinetic friction between the two surfaces is 0.50. Find the acceleration of the block.



109. As shown below, the mass of block 1 is $m_1 = 4.0\,\text{kg}$, while the mass of block 2 is $m_2 = 8.0\,\text{kg}$. The coefficient of friction between m_1 and the inclined surface is $\mu_k = 0.40$. What is the acceleration of the system?

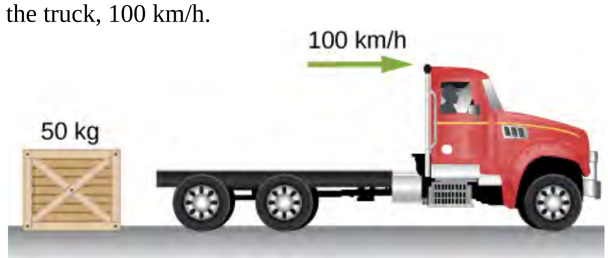


110. A student is attempting to move a 30-kg mini-fridge into her dorm room. During a moment of inattention, the mini-fridge slides down a 35 degree incline at constant speed when she applies a force of 25 N acting up and parallel to the incline. What is the coefficient of kinetic friction between the fridge and the surface of the incline?

111. A crate of mass 100.0 kg rests on a rough surface inclined at an angle of $37.0°$ with the horizontal. A massless rope to which a force can be applied parallel to the surface is attached to the crate and leads to the top of the incline. In its present state, the crate is just ready to slip and start to move down the plane. The coefficient of friction is 80% of that for the static case. (a) What is the coefficient of static friction? (b) What is the maximum force that can be applied upward along the plane on the rope and not move the block? (c) With a slightly greater applied force, the block will slide up the plane. Once it begins to move, what is its acceleration and what reduced force is necessary to keep it moving upward at constant speed? (d) If the block is given a slight nudge to get it started down the plane, what will be its acceleration in that direction? (e) Once the block begins to slide downward, what upward force on the rope is required to keep the block from accelerating downward?

112. A car is moving at high speed along a highway when the driver makes an emergency braking. The wheels become locked (stop rolling), and the resulting skid marks are 32.0 meters long. If the coefficient of kinetic friction between tires and road is 0.550, and the acceleration was constant during braking, how fast was the car going when the wheels became locked?

113. A crate having mass 50.0 kg falls horizontally off the back of the flatbed truck, which is traveling at 100 km/h. Find the value of the coefficient of kinetic friction between the road and crate if the crate slides 50 m on the road in coming to rest. The initial speed of the crate is the same as the truck, 100 km/h.



114. A 15-kg sled is pulled across a horizontal, snow-covered surface by a force applied to a rope at 30 degrees with the horizontal. The coefficient of kinetic friction between the sled and the snow is 0.20. (a) If the force is 33 N, what is the horizontal acceleration of the sled? (b) What must the force be in order to pull the sled at constant velocity?

115. A 30.0-g ball at the end of a string is swung in a vertical circle with a radius of 25.0 cm. The rotational velocity is 200.0 cm/s. Find the tension in the string: (a) at the top of the circle, (b) at the bottom of the circle, and (c) at a distance of 12.5 cm from the center of the circle $(r = 12.5\,\text{cm})$.

116. A particle of mass 0.50 kg starts moves through a circular path in the *xy*-plane with a position given by $\vec{\mathbf{r}}\,(t) = (4.0\cos 3t)\,\hat{\mathbf{i}} + (4.0\sin 3t)\,\hat{\mathbf{j}}$ where r is in meters and t is in seconds. (a) Find the velocity and acceleration vectors as functions of time. (b) Show that the acceleration vector always points toward the center of the circle (and thus represents centripetal acceleration). (c) Find the centripetal force vector as a function of time.

117. A stunt cyclist rides on the interior of a cylinder 12 m in radius. The coefficient of static friction between the tires and the wall is 0.68. Find the value of the minimum speed for the cyclist to perform the stunt.

118. When a body of mass 0.25 kg is attached to a vertical massless spring, it is extended 5.0 cm from its unstretched length of 4.0 cm. The body and spring are placed on a horizontal frictionless surface and rotated about the held end of the spring at 2.0 rev/s. How far is the spring stretched?

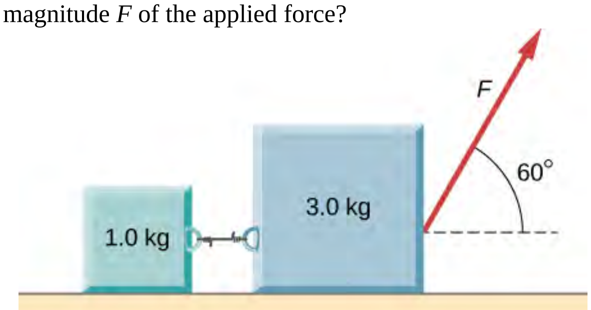
119. Railroad tracks follow a circular curve of radius 500.0 m and are banked at an angle of $5.00°$. For trains of what speed are these tracks designed?

120. A plumb bob hangs from the roof of a railroad car. The car rounds a circular track of radius 300.0 m at a speed of 90.0 km/h. At what angle relative to the vertical does the plumb bob hang?

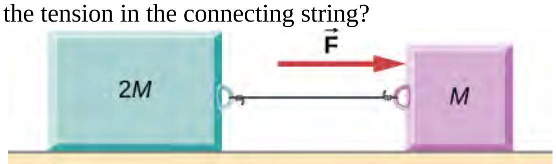
121. An airplane flies at 120.0 m/s and banks at a $30°$ angle. If its mass is 2.50×10^3 kg, (a) what is the magnitude of the lift force? (b) what is the radius of the turn?

122. The position of a particle is given by $\vec{\mathbf{r}}\,(t) = A\left(\cos \omega t\,\hat{\mathbf{i}} + \sin \omega t\,\hat{\mathbf{j}}\right)$, where ω is a constant. (a) Show that the particle moves in a circle of radius A. (b) Calculate $d\,\vec{\mathbf{r}}\,/dt$ and then show that the speed of the particle is a constant A_ω. (c) Determine $d^2\,\vec{\mathbf{r}}\,/dt^2$ and show that a is given by $a_\text{c} = r\omega^2$. (d) Calculate the centripetal force on the particle. [*Hint*: For (b) and (c), you will need to use $(d/dt)(\cos \omega t) = -\omega \sin \omega t$ and $(d/dt)(\sin \omega t) = \omega \cos \omega t$.

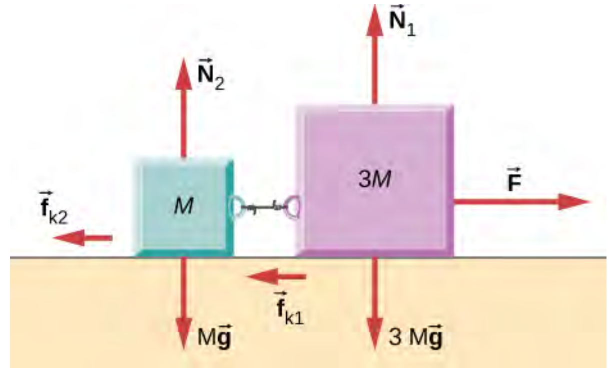
123. Two blocks connected by a string are pulled across a horizontal surface by a force applied to one of the blocks, as shown below. The coefficient of kinetic friction between the blocks and the surface is 0.25. If each block has an acceleration of $2.0\,\text{m/s}^2$ to the right, what is the magnitude F of the applied force?



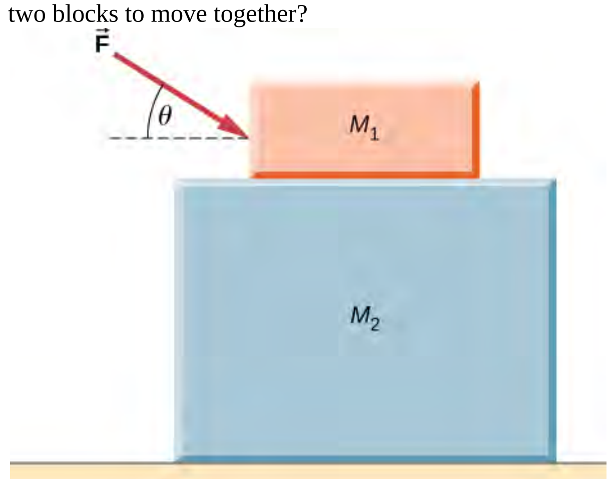
124. As shown below, the coefficient of kinetic friction between the surface and the larger block is 0.20, and the coefficient of kinetic friction between the surface and the smaller block is 0.30. If $F = 10\,\text{N}$ and $M = 1.0\,\text{kg}$, what is the tension in the connecting string?



125. In the figure, the coefficient of kinetic friction between the surface and the blocks is μ_k. If $M = 1.0\,\text{kg}$, find an expression for the magnitude of the acceleration of either block (in terms of F, μ_k, and g).

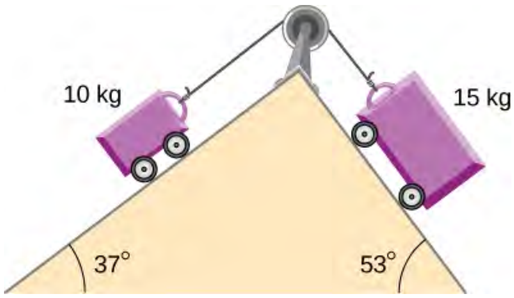


126. Two blocks are stacked as shown below, and rest on a frictionless surface. There is friction between the two blocks (coefficient of friction μ). An external force is applied to the top block at an angle θ with the horizontal. What is the maximum force F that can be applied for the two blocks to move together?



127. A box rests on the (horizontal) back of a truck. The coefficient of static friction between the box and the surface on which it rests is 0.24. What maximum distance can the truck travel (starting from rest and moving horizontally with constant acceleration) in 3.0 s without having the box slide?

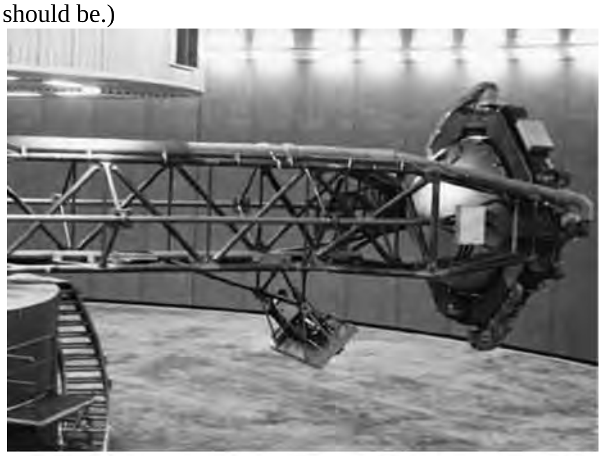
128. A double-incline plane is shown below. The coefficient of friction on the left surface is 0.30, and on the right surface 0.16. Calculate the acceleration of the system.



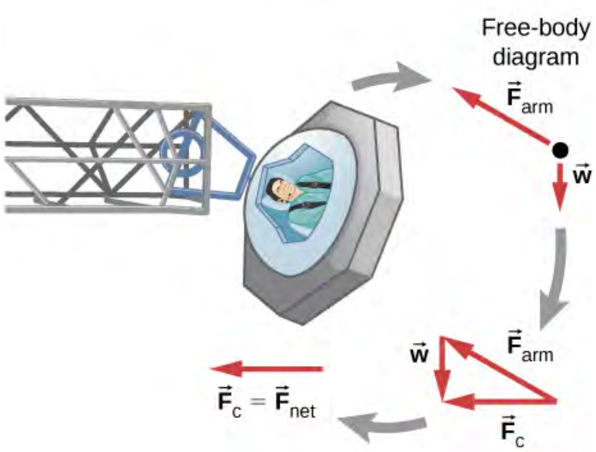
CHALLENGE PROBLEMS

129. In a later chapter, you will find that the weight of a particle varies with altitude such that $w = \frac{mgr_0{}^2}{r^2}$ where r_0 is the radius of Earth and *r* is the distance from Earth's center. If the particle is fired vertically with velocity v_0 from Earth's surface, determine its velocity as a function of position *r*. (*Hint:* use $a\,dr = v\,dv,$ the rearrangement mentioned in the text.)

130. A large centrifuge, like the one shown below, is used to expose aspiring astronauts to accelerations similar to those experienced in rocket launches and atmospheric reentries. (a) At what angular velocity is the centripetal acceleration $10g$ if the rider is 15.0 m from the center of rotation? (b) The rider's cage hangs on a pivot at the end of the arm, allowing it to swing outward during rotation as shown in the bottom accompanying figure. At what angle θ below the horizontal will the cage hang when the centripetal acceleration is $10g$? (*Hint:* The arm supplies centripetal force and supports the weight of the cage. Draw a free-body diagram of the forces to see what the angle θ should be.)



(a)



(b)

131. A car of mass 1000.0 kg is traveling along a level road at 100.0 km/h when its brakes are applied. Calculate the stopping distance if the coefficient of kinetic friction of the tires is 0.500. Neglect air resistance. (*Hint:* since the distance traveled is of interest rather than the time, x is the desired independent variable and not t. Use the Chain Rule to change the variable: $\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$.)

132. An airplane flying at 200.0 m/s makes a turn that takes 4.0 min. What bank angle is required? What is the percentage increase in the perceived weight of the passengers?

133. A skydiver is at an altitude of 1520 m. After 10.0 seconds of free fall, he opens his parachute and finds that the air resistance, F_{D}, is given by the formula $F_{\mathrm{D}} = -bv$, where b is a constant and v is the velocity. If $b = 0.750$, and the mass of the skydiver is 82.0 kg, first set up differential equations for the velocity and the position, and then find: (a) the speed of the skydiver when the parachute opens, (b) the distance fallen before the parachute opens, (c) the terminal velocity after the parachute opens (find the limiting velocity), and (d) the time the skydiver is in the air after the parachute opens.

134. In a television commercial, a small, spherical bead of mass 4.00 g is released from rest at $t = 0$ in a bottle of liquid shampoo. The terminal speed is observed to be 2.00 cm/s. Find (a) the value of the constant b in the equation $v = \frac{mg}{b}(1 - e^{-bt/m})$, and (b) the value of the resistive force when the bead reaches terminal speed.

135. A boater and motor boat are at rest on a lake. Together, they have mass 200.0 kg. If the thrust of the motor is a constant force of 40.0 N in the direction of motion, and if the resistive force of the water is numerically equivalent to 2 times the speed v of the boat, set up and solve the differential equation to find: (a) the velocity of the boat at time t; (b) the limiting velocity (the velocity after a long time has passed).